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THE PRINCIPLE OF LIMITING ABSORPTION FOR THE NON-SELFADJOINT SCHRÖDINGER OPERATOR IN R^2

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Introduction

The present paper is a continuation of [3] and is devoted to extending the results obtained in [3] to the non-selfadjoint Schrödinger operator in $\boldsymbol{R}^2$.

In the paper [3] we considered the non-selfadjoint Schrödinger operator

$$(0.1) \qquad L = -\sum_{j=1}^{N}\left(\frac{\partial}{\partial x_j}+ib_j(x)\right)^2+Q(x)$$

in $\boldsymbol{R}^N$, where N is a positive integer such that $N\neq 2$, and the complex-valued function $Q(x)$ and the real-valued functions $b_j(x)$ $(j=1, 2, \cdots, N)$ are assumed to satisfy some asymptotic conditions at infinity. Among others we have shown the following: Let us define a Hilbert space $L_{2,\beta}=L_{2,\beta}(\boldsymbol{R}^N)$ $(\beta\in\boldsymbol{R})$ by

$$(0.2) \qquad L_{2,\beta} = \{f(x)/(1+|x|)^\beta f(x)\in L_2(\boldsymbol{R}^N)\}$$

with its inner product

$$(0.3) \qquad (f, g)_\beta = \int_{\boldsymbol{R}^N}(1+|x|)^{2\beta}f(x)\overline{g(x)}\,dx$$

and norm

$$(0.4) \qquad \|f\|_\beta = [(f, f)_\beta]^{1/2}\,.$$

If $\kappa\in\boldsymbol{C}_+=\{\kappa\in\boldsymbol{C}/\kappa\neq 0 \text{ and } \operatorname{Im}\kappa\geqq 0\}$ does not belong to an exeptional set which is called the set of the singular points of L, then the operator $(L-\kappa^2)^{-1}$ is well-defined as a bounded linear operator from $L_{2,(1+\varepsilon)/2}$ into $L_{2,-(1+\varepsilon)/2}$ $(\varepsilon>0)$ with the estimate

$$(0.5) \qquad \|(L-\kappa^2)^{-1}\| = O(|\kappa|^{-1}) \qquad (|\kappa|\to\infty)\,.$$

Here $u=(L-\kappa^2)^{-1}f\in L_{2,-(1+\varepsilon)/2}$ $(f\in L_{2,(1+\varepsilon)/2})$ is a unique solution of the equation

$$(0.6) \qquad (L-\kappa^2)u = f$$

with a sort of "radiation condition", and $\|(L-\kappa^2)^{-1}\|$ means the operator norm