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AN APPROXIMATE POSITIVE PART OF A SELF-ADJOINT PSEUDO-DIFFERENTIAL OPERATOR II

DAISUKE FUJIWARA

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1. Introduction

Let $P=P(x, D)$ be a self-adjoint pseudo-differential operator with the symbol $p(x, \xi)$ in the class $S^1_{1,0}$ of Hörmander. The positive part of P is defined by

$$P^+ = \int_0^\infty \lambda\, dE(\lambda)\,,$$

where $dE(\lambda)$ is the spectral measure of P. We shall be concerned with the following question: To what extent the correspondence; $u \to P^+u$ can be localized? We shall prove a localization principle for the operator P^+ which is analogous to Theorem 6.3 of Hörmander [5]. If we combine this with our previous discussions in [2], we can explicitly construct an operator B such that we have estimate

$$|((A^+ - B)u, v)| \leqq C\|u\|_{1/6}\|v\|_{1/6}\,,$$

where u and v are arbitrary functions in $\mathcal{D}(\boldsymbol{R}^n)$ and C is a positive constant independent of u and v.

2. Localized operators

Let us repeat our notations. $p(x, \xi)$ is a function in the class $S^1_{1,0}$ which vanishes unless x lies in a compact set K in $\boldsymbol{R}^n$. We treat pseudo-differential operator $P(x, D)$ defined as

$$P(x, D)u(x) = (2\pi)^{-n} \iint_{\boldsymbol{R}^{2n}} p(x, \xi)u(y)e^{i\langle x-y\rangle\cdot\xi}\, dy\, d\xi\,. \tag{2.1}$$

We assume that $P=P(x, D)$ is self-adjoint in Hilbert space $L^2(\boldsymbol{R}^n)$.

Now we make use of the partition of unity of Hörmander [5]. Let $g_0=0$, g_1, g_2, $\cdots$ be the unit lattice points in $\boldsymbol{R}^n$. Then $\boldsymbol{R}^n$ is covered by open cubes of side 2 with center at these points. Let $\Theta(x)$ be a non-negative C_0^∞ function which equals 1 on $|x_j| \leqq 1$ and 0 outside $|x_j| \leqq 3/2$, $j=1, 2, 3, \cdots, n$. We set