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AN APPROXIMATE POSITIVE PART OF A SELF-ADJOINT PSEUDO-DIFFERENTIAL OPERATOR I

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1. Introduction

Among many problems concerning pseudo-differential operators, one of the most interesting problem is "to what extent does the symbol function $p(x, \xi)$ describe the spectral properties of an operator $p(x, D)$?" Motivation of this paper comes from this problem.

Actually what we do in this note is the following: Assume that $P=p(x, D)$ is a self-adjoint pseudo-differential operator of class $L^0_{1,0}$ of Hörmander [4]. Then starting from its principal symbol, we explicitly construct self-adjoint operators P^+, P^-, R, F^+ and F^- with the following properties;

(i) $F^+ + F^- = Id$.

(ii) $P = P^+ - P^- + R$.

(iii) P^+, P^- and F^+, F^- are non-negative self-adjoint operators.

(iv) We have the following estimates;

$$|(P^+F^-u, F^\pm v)| \leqq C\|u\|_{-1/3}\|v\|_{-1/3},$$
$$|(P^-F^+u, F^\pm v)| \leqq C\|u\|_{-1/3}\|v\|_{-1/3},$$
$$|(Ru, v)| \leqq C\|u\|_{-1/3}\|v\|_{-1/3},$$
for any $u, v \in C_0^\infty(\boldsymbol{R}^n)$.

Theorem I gives more precise statement. Proof is found in §5 and §6.

If the principal symbol does not change sign, the problem has been settled. In fact strong Gårding inequality [3], [6] means that we can take $P^-=0$, $F^-=0$ and that R satisfies stronger inequality

$$|(Ru, v)| \leqq C\|u\|_{-1/2}\|v\|_{-1/2}.$$

However our result seems new if the principal symbol changes sign. Difficulty arises at the point of characteristics of the operator $p(x, D)$. The operator F^+ and F^- are closely related to location of characteristics of $p(x, D)$. This is discussed in §7.

1) As to general theory of pseudo-differential operators. See [1], [2], [5] and [7].