

ON THE CAUCHY PROBLEM FOR PARABOLIC PSEUDO-DIFFERENTIAL EQUATIONS

MICHIHIRO NAGASE

(Received September 26, 1973)

1. Introduction

In the recent paper [8] S. Kaplan has obtained an analogue of Gårding's inequality for parabolic differential operators and applied it to a Hilbert space treatment of the Cauchy problem. D. Ellis [3] has extended those results to higher order parabolic differential operators (see also [4]). On the other hand in [13] the author has studied a Hilbert space treatment of the Cauchy problem for parabolic pseudo-differential equations and generalized the results of S. Kaplan [8].

In the present paper we shall study the Cauchy problem for higher order parabolic pseudo-differential equations of the form

$$Lu = D_t^k u(t, x) + \sum_{j=1}^k p_j(t, X, D_x) D_t^{k-j} u(t, x) = f(t, x)$$

where $p_j(t, x, \xi)$ are symbols of the class $S_{\rho, \lambda}^m$ introduced in [11] and [12]. We need not assume that the basic weight function $\lambda(\xi)$ tends to infinity as $|\xi| \rightarrow \infty$. Therefore the theory can be applied to more general classes of operators (including difference operators) than the class of usual parabolic differential operators.

In section 2 we give definitions and lemmas for pseudo-differential operators. In section 3 the algebras and L^2 -theory are stated. The L^2 -continuity of pseudo-differential operators has been studied in many papers (see for example, Calderón and Vaillancourt [1], [2], Hörmander [7] and Kumano-go [10]). In the present paper the L^2 -continuity theorem by Calderón and Vaillancourt in [1] plays an essential role. In section 4 we define the space $H_{r,s}(\Omega)$ which is needed to study the Cauchy problem. In section 5 we derive energy inequalities for the parabolic system which is associated with a higher order parabolic pseudo-differential operator. These energy inequalities are very similar to those of D. Ellis [3] and [4]. To obtain the energy inequalities the idea of double symbols of pseudo-differential operators is very important. In section 6, using the results in section 4 and 5, we discuss a Hilbert space treatment of Cauchy problem for parabolic systems. In section 7 finally we state the main results for the Cauchy problem for higher order parabolic pseudo-differential equations.