Taniguchi, K.
Osaka J. Math.
11 (1974), 221–238

ON THE HYPOELLIPTICITY AND THE GLOBAL ANALYTIC-HYPOELLIPTICITY OF PSEUDO-DIFFERENTIAL OPERATORS

KAZUO TANIGUCHI

(Received September 14, 1973)

Introduction

In the recent paper [13] Kumano-go and Taniguchi have studied by using oscillatory integrals when pseudo-differential operators in R^n are Fredholm type and examined whether or not the operators $L_k(x, D_x, D_y)=D_x+ix^kD_y$ in Mizohata [15] and $L_\pm(x, D_x, D_y)=D_x\pm ixD_y^2$ in Kannai [6] are hypoelliptic by a unified method. In the present paper we shall give the detailed description for results obtained in [13] and study the hypoellipticity for the operator of the form $L=\sum_{|\alpha:\mathfrak{m}|+|\alpha':\mathfrak{m}'|\leqq 1} a_{\alpha\alpha'\gamma\gamma'}x^\gamma \tilde{y}^{\gamma'}D_x^\alpha D_y^{\alpha'}$ with semi-homogeneity in $(x, \tilde{y}, D_x, D_y)$ by deriving the similar inequality to that of Grushin [4] for the elliptic case. Then we can treat the semi-elliptic case as well as the elliptic case. We shall also give a theorem on the global analytic-hypoellipticity of a non-elliptic operator, and applying it give a necessary and sufficient condition for the operator $L(x, D_x, D_y)$ to be hypoelliptic, when the coefficients of L are independent of $\tilde{y}^{\gamma'}$ (see Theorem 3.1).

In Section 1 we shall describe pseudo-differential operators of class $S^m_{\lambda,\rho,\delta}$ which is defined by using a basic weight function $\lambda=\lambda(x, \xi)$ varying in x and ξ (cf. [13] and also [1]). In Section 2 we shall study the global analytic-hypoellipticity of a non-elliptic pseudo-differential operator and give an example which indicates that the condition (2.3) is necessary in general. In Section 3 we shall consider the local hypoellipticity for the operator L and give some examples.

The author wishes to thank Prof. H. Kumano-go for suggesting this problem and his helpful advice.

1. Algebras and L^2-boundedness

DEFINITION 1.1. For $-\infty<m<\infty$, $0\leqq\delta<1$ and a sequence $\tilde{\tau}$; $0\leqq\tau_0\leqq\tau_1\leqq\cdots$ we define a Fréchet space $\mathcal{A}^m_{\delta,\tilde{\tau}}$ by the set of C^∞-functions $p(\xi, x)$ in $R^{2n}_{\xi,x}$ for which each semi-norm