ORIENTED AND WEAKLY COMPLEX BORDISM
OF FREE METACYCLIC ACTIONS

KATSUYUKI SHIBATA

Abstract. Oriented and weakly complex bordism modules of free metacyclic actions are determined up to the Kasparov formula which describes the bordism classes of generalized lens spaces in terms of a linear combination of those of the standard lens spaces. In the oriented case for $p=2$ (the dihedral case), the module structure is particularly simple because the corresponding Kasparov formula reduces to the multiplication by ± 1. We also compute the abelian group structure of these bordisms in case $p \geqslant 2$ a prime and $q \geqslant 3$ an odd prime. Of independent interest is the canonical projections defined on these bordism modules which select a direct summand with one generator in each $2pj-1$ dimension $(j=1, 2, \cdots)$.

1. Introduction.

Let $Z_{q,p}$ be the metacyclic group

$$Z_{q,p} = \{x, y \,|\, x^q = y^p = 1, \, yxy^{-1} = x^r\}$$

where $p \geqslant 2$ is a prime integer, $q \geqslant 3$ is an odd integer and r is a primitive p-th root of 1 mod q such that $(r-1, q)=1$. (So $r \equiv -1$ mod q when $p=2$.) By virtue of Fermat's theorem, these conditions imply $(p, q)=1$.

Obviously there is an exact sequence

$$1 \longrightarrow Z_q \xrightarrow{\ i\ } Z_{q,p} \underset{s}{\overset{\pi}{\rightleftarrows}} Z_p \longrightarrow 1$$

with s a corss-section defined by $s(\bar{y})=y$.

Kamata—Minami [3] determined the additive structure of the weakly complex reduced bordism group of the free dihedral group actions $\tilde{\Omega}_m^U(Z_{q,2})$ in case q is an odd prime. Here we generalize their results to the cases for the oriented and weakly complex bordism modules $\tilde{\Omega}_*^{SO}(Z_{q,p})$ and $\tilde{\Omega}_*^U(Z_{q,p})$ of the free metacyclic actions.

For the basic notations and prerequisites, we refer the reader to the introductory part and §1 of Kamata—Minami [3].