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INVOLUTIONS AND CIRCLE ACTIONS WHICH BORD

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In [7; §7] Fuichi Uchida demonstrates that the Thom-Gysin sequence enables one to determine that any smooth principal circle action on $S^{2m+1}\times S^{2n+1}$ bords as a free S^1 action. In this paper similar results are proved for free involutions, Uchida's results on circle actions extended in several directions, and some results are elicited on the bordism of arbitrary involutions on products of two spheres.

Denote by $\hat{N}_*(G)$ and $\hat{\Omega}_*(G)$ respectively the bordism of smooth principal G actions on closed smooth mainfolds and the bordism of smooth principal orientation preserving G actions on closed smooth oriented mainfolds. In section 1 it is shown that

Theorem 1.3. *If T is a smooth fixed point free involution on the product of two spheres $S^m\times S^n$, then $[S^m\times S^n, T]$ bords in $\hat{N}_*(Z_2)$.*

In section 2 it is shown that

Theorem 2.1. *Any smooth principal circle action on $S^m\times S^n$ bords in $\hat{N}_*(S^1)$.*

Note that Uchida's result [7; th. 7.3.] is a corollary to this theorem. Also in section 2 one finds

Theorem 2.4. *Any principal circle action on $(S^{2j+1})^{2k}$ bords in $\hat{N}_*(S^1)$.*

Theorem 2.5. *If $RP(n_1)\times\cdots\times RP(n_r)$ is a product of real projective spaces so that at least two of the n_j are odd, then any free circle action on $RP(n_1)\times\cdots\times RP(n_r)$ bords in $\hat{N}_*(S^1)$.*

Further, there are corollaries to each of these results giving modified oriented analogues.

In section 3 the bordism of arbitrary smooth involutions on products of spheres is examined. Let $N_*(Z_2)$ be the bordism of unrestricted smooth involutions. Since there is an injection from $N_*(Z_2)$ into $\oplus N_*(BO_k)$ given by classifying the normal bundle to the fixed set [3; §28], it suffices to consider the bordism of the fixed set and its normal bundle. Hence an involution on a single sphere