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# CONTINUOUS MAPS OF MANIFOLDS WITH INVOLUTION II

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## Introduction

Let $N$ and $M$ be closed manifolds on each of which an involution is given, and assume that the involution on $N$ is free. In the previous paper [9], the author defined the equivariant Lefschetz class of a continuous map $f: N \to M$, and treated the class in the case when the involution on $M$ is also free. The present paper is concerned with the equivariant Lefschetz class in the case when the involution on $M$ is trivial. As applications, we show generalizations of the Borsuk-Ulam theorem and also theorem of group action on manifolds.

Compared with the previous case, the expression of the equivariant Lefschetz class in the present case is rather complicated, and the Wu classes of mainfolds and the operations of Breden [1] appear in it. Some properties of the semi-characteristic of manifolds are also needed in our applications. These are prepared in §1 and §2 (see also Appendix).

Throughout this paper, the homology and cohomology with coefficients in $Z_2$ are to be understood. For brevity, manifolds and actions on them are assumed to be differentiable.

## 1. Semicharacteristic of manifolds with involution

If $M$ is a closed manifold such that the dimension of the vector space $H^*(M)$ is even, an integer mod 2 given by

$$\hat{\chi}(M) = \frac{1}{2} \dim H^*(M) \qquad \bmod 2$$

is called the *semicharacteristic* of $M$. If $N$ is a closed manifold with a free involution, it is known that dim $H^*(N)$ is even (see [1], [9]). In this section we shall consider the semicharacteristic of closed manifolds with involution.

(1.1) **Proposition.** *Let $W$ be a compact $(n+1)$-dimensional manifold with boundary $\partial W$, and assume that $W$ has a free involution $T$. Then we have $\hat{\chi}(\partial W)=0$.*