Suzuki, S.
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ON A COMPLEXITY OF A SURFACE IN 3-SPHERE

Dedicated to Professor Ralph H. Fox for his 60th birthday

SHIN'ICHI SUZUKI

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0. Introduction

Throughout this paper we shall only be concerned with the combinatorial category, consisting of simplicial complexes and piecewise-linear maps. It is the purpose of the paper to prove intuitively obvious topological theorems which are interesting in the Morse theory of 3-manifolds. The theorems concern "knot types" of embeddings of a closed (=compact, without boundary), connected and orientable surface M_p of genus p into the 3-dimensional sphere S^3.

As widely known, a surface M_p in S^3, denoted by $(M_p \subset S^3)$, is obtained from some 2-spheres by adding handles, Fox [3] and Homma [5]. Using the fact, we shall define a complexity $\langle s, t\rangle$, a pair of natural numbers, for the knot type of the $(M_p \subset S^3)$ in §1. After establishing a canonical representative for the knot type of $(M_p \subset S^3)$ in §2, we first consider some non-existence results in §3. In §4 and §5, we construct some pairs $(M_p \subset S^3)$'s for some complexities $\langle s, t\rangle$'s.

In the paper, homeomorphism is denoted by $\cong$, while $\simeq$ and $\sim$ refer to homotopy and homology, respectively. ∂X, cl (X) and $^\circ X$ denote, respectively, the boundary, the closure and the interior of a manifold X. By D^n and S^{n-1} we shall denote the standard n-cell and the standard (n−1)-sphere ∂D^n, respectively, and particularly, $D^1=[-1, 1]$.

1. Definitions and notation

First let us explain several definitions and notation, and formulate our main theorem.

In general, we shall denote by M a compact orientable surface, and $^\#(M)$ and $g(M)$ stand for the number of connected components of M and the total genus of M, respectively.

We shall say that a submaifold X of a manifold Y is *properly embedded* (or simply *proper*) if $X \cap \partial Y = \partial X$.

By $(M \subset M^3)$ we denote a pair of mainfolds such that a 3-manifold M^3 and