Mimura, M., Mukai, J. and Nishida, G.
Osaka J. Math.
11 (1974), 105-111

REPRESENTING ELEMENTS OF STABLE HOMOTOPY GROUPS BY SYMMETRIC MAPS

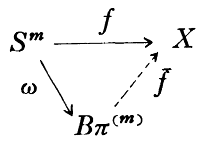
MAMORU MIMURA, JUNO MUKAI AND GORO NISHIDA

(Received June 14, 1973)

0. Introduction

Let S^m be the unit m-sphere. Let p be a prime and π the cyclic group of order p. Denote by $B\pi^{(r)}$ the r-skeleton of the classifying space $B\pi$. Recall that $B\pi$ is the infinite real projective space for $p=2$ and the infinite lens space for $p>2$. Let X be a space. Let m be a positive integer for the case $p=2$ and m an odd integer for the case $p>2$. Then a map $f: S^m \to X$ is called *symmetric* if there exists a map $\bar{f}: B\pi^{(m)} \to X$ such that the following diagram is commutative:

(1)



, where $\omega: S^m \to B\pi^{(m)}$ is the canonical projection.

An element of the homotopy group $\pi_m(X)$ is called *symmetric* if it is represented by a symmetric map. For $p=2$, the definition of a symmetric map is due to J. H. C. Whitehead [14], in which he showed that if an essential element of $\pi_m(S^{m-1})$ is symmetirc, then $m \equiv 3 \bmod 4$. Some results about the symmetricity of the elements of $\pi_m(X)$ are found in [4], [8], [10], [21] and [13].

Let X be an $(l-1)$-connected, finite CW-complex. Then our purpose is to show the following

Theorem 1. *Every element of* $\pi_m(X)$ *is symmetric for any* m *satisfying* $2 \dim X-l<m<2l-2$ *and*

i) $m \equiv -1 \bmod 2^{\phi(k+1)}$ *for* $p=2$,

ii) $m \equiv -1 \bmod 2p^{[(k+1)/2(p-1)]}$ *for* $p>2$,

where $k=m-l$, $\phi(s)$ *is the number of integers* i *such that* $0<i \leqq s$ *and* $i \equiv 0, 1, 2$ *or* $4 \bmod 8$ *and* $[s]$ *indicates the integer part of a rational* s.

Corollary 2. *For an arbitrary* $k>0$, *every element of the* k*-stem of the stable*