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SOME APPLICATIONS OF THE ROTHENBERG-STEENROD SPECTRAL SEQUENCE

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1. Introduction. In the study of principal actions of a group G, a fundamental role is played by the classifying space B_G. Thus it is natural to seek algebraic invariants which describe the geometrical properties of these spaces. For the purpose of studying their homology and cohomology, Rothenberg and Steenrod [15] introduced a variation of the Eilenberg-Moore spectral sequence and gave several applications. Hodgkin [11] and Anderson and Hodgkin [2] recast the cohomological form of this spectral sequence into K^*-theory and used it to study the K^*-theory of Lie groups and Eilenberg-MacLane spaces.

It is our purpose here to extend the homological form of the spectral sequence to arbitrary multiplicative generalized homology theories and give some brief applications. Since the constructions require a Künneth isomorphism, we must introduce cyclic groups of coefficients and investigate the existence of associated multiplicative structures. This is done in §2 and follows the corresponding constructions of Araki and Toda [3] for cohomology. In §3 the spectral sequence is described and the E^2-stage and edge homomorphism are identified.

The applications are given in §4. These include the computation of the K_*-groups of certain Eilenberg-MacLane spaces, using results of Anderson and Hodgkin [2]. The implications of these computations in complex bordism are noted briefly. Finally we give the following generalization of a theorem of Borel [5]: If h_* is a multiplicative homology theory, p is a prime, $h_*(pt.; Z_p)=R$ is zero in odd dimensions and G is a group having $h_*(G; Z_p)$ an exterior algebra over R on a finite number of odd dimensional generators, then $h_*(B_G; Z_p)$ is a modified polynomial algebra over R on corresponding generators of one dimension higher.

We assume throughout that spaces are in the category A of spaces having the homotopy type of a CW complex with finite skeleta and that all homology theories are additive. It is a pleasure to acknowledge recent conversations with Gary Hamrick on this and related subjects.

2. Multiplicative homology theories. Let h_* be a generalized homology