Fukushima, M.
Osaka J. Math.
11 (1974), 73–85

ON THE SPECTRAL DISTRIBUTION OF A DISORDERED SYSTEM AND THE RANGE OF A RANDOM WALK

Masatoshi FUKUSHIMA

(Received July 17, 1973)

1. Introduction

Consider the ν-dimensional lattice Z^ν. We define a second order difference operator H^0 by

$$(H^0 u)(a) = \frac{\sigma^2}{2}\sum_{i=1}^{\nu}\{u(a_1, \cdots, a_i-1, \cdots, a_\nu)-2u(a)+u(a_1, \cdots, a_i+1, \cdots, a_\nu)\}\ ,$$
$$a\in Z^\nu,\ u\in C_0(Z^\nu)\,,$$

where σ is a positive constant and $C_0(Z^\nu)$ is the space of functions on Z^ν with finite supports. Let $\{q(a)=q(a, \omega);\ a\in Z^\nu\}$ be a family of independent, identically distributed non-negative random variables defined on some probability space $(\Omega, \mathcal{B}, P)$. We are then concerned with the difference operator H^ω depending on the random parameter $\omega\in\Omega$:

$$(1)\qquad (H^\omega u)(a) = (H^0 u)(a)-q(a, \omega)u(a)\,, \qquad a\in Z^\nu\,.$$

The operator $-H^\omega$, considered as a linear transform over $C_0(Z^\nu)$, is a non-negative definite symmetric operator on $L^2(Z^\nu)$ and has a unique self-adjoint extension $-\bar{H}^\omega$. Express $-\bar{H}^\omega$ as $-\bar{H}^\omega=\int_{[0,\infty)} x dE_x^\omega$ by the associated spectral family $\{E_x^\omega, -\infty<x<\infty\}$ and put $\rho^\omega(x)=(E_x^\omega I_0, I_0)$, where $(\ ,\ )$ is the L^2-inner product and $I_a(a')=\delta_{aa'}$, $a, a'\in Z^\nu$.

Denote by $\langle\ \rangle$ the expectation with respect to the probability measure P and set

$$(2)\qquad \rho(x) = \langle \rho^\cdot(x)\rangle, \ -\infty<x<\infty\,.$$

$\rho(x)$ is a probability distribution function vanishing on $(-\infty, 0)$. We call this the *spectral distribution function* associated with the ensemble of operators $\{H^\omega, \omega\in\Omega\}$ or rather with the disordered dynamical system governed by H^ω's (*e.g.* a tight binding electron model [4]).

Our main aim is to show in §4 the following asymptotic behaviours of $\rho(x)$ near the origin.