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ON THE GENERALIZED SOLITON SOLUTIONS OF THE MODIFIED KORTEWEG-DE VRIES EQUATION

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In this paper, we discuss the asymptotic property of the generalized soliton solutions of the modified Korteweg-de Vries (*K-d*v) equation

$$(1) \qquad v_t+6v^2v_x+v_{xxx}=0\,, \qquad -\infty<x,\ t<\infty$$

where v_t and v_x denote partial derivatives of $v=v(x, t)$ with respect to t and x respectively. This equation gives one of the simplest modifications of the *K-dV* equation

$$u_t-6uu_x+u_{xxx}=0\,, \qquad -\infty<x,\ t<\infty\,.$$

Both of the *K-dV* equation and the modified *K-dV* equation are known to have progressive wave solutions;

$$u(x, t)=-2^{-1}c\ \mathrm{sech}^2\,(2^{-1}c^{1/2}(x-ct-\delta))\,, \qquad c>0$$

for the *K-dV* equation,

$$(2) \qquad v(x, t)=\pm c^{1/2}\ \mathrm{sech}\,(c^{1/2}(x-ct-\delta))\,, \qquad c>0$$

for the modified *K-dV* equation.

Each of such solutions is called a solitary wave solution or a soliton on account of its shape.

On the other hand, Gardner, Greene, Kruskal and Miura [1] have related the solution $u(t)=u(x, t)$ of the *K-dV* equation to the scattering theory of the one dimensional Schrödinger operator with the potential $u(t)$ and found that discrete eigenvalues are invariants and the reflection coefficient and normalization coefficients vary exponentially with respect to t (see also Lax [3]). Here a soliton of the *K-dV* equation is characterized as the solution with one discrete eigenvalue and the zero reflection coefficient. Furthermore, the reflectionless potential with N discrete eigenvalues can be written in closed form in terms of exponentials by the method of Kay and Moses [2] for each t. These potentials are called N-tuple wave solutions. N-tuple wave solution