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KORTEWEG-DE VRIES EQUATION: CONSTRUCTION OF SOLUTIONS IN TERMS OF SCATTERING DATA

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In this paper we show that the solution of the initial value problem for the Korteweg-de Vries (KdV) equation

$$(1) \qquad u_t - 6uu_x + u_{xxx} = 0 \qquad u = u(t) = u(x, t) \quad (-\infty < x, t < \infty)$$

can be constructued by a method suggested in Gardner, Greene, Kruskal and Miura (GGKM) [2].

GGKM have associated one dimensional Schrödinger operator

$$L_{u(t)} = -(d/dx)^2 + u(x, t)$$

to a solution of (1). They have found remarkable facts concerning the time dependence of scattering data of $L_{u(t)}$: eigen-values are invariants and reflection coefficient $r(\xi, t)$ of $L_{u(t)}$ is given by

$$r(\xi, t) = r(\xi, 0) \exp(8i\, \xi^3 t) \qquad -\infty < \xi < \infty$$

etc. In [2], the authors also have pointed out that application of inverse scattering theory leads to certain explicit realization of the solution. It is not unexpected that use of inverse scattering theory may even lead to the construction of general solutions of the initial value problem. However the existence of solutions of nice properties has been a priori assumed in [2] and this possibility has not been explored.

The results of the present paper have been announced in [7]. GGKM's method has been also formulated in Faddeev and Zakharov [8] in a different form.

In §1 we describe results from scattering theory of one dimensional Schrödinger operator. The materials which connect the KdV equation and scattering theory are described in §2. Then in §3 analytical properties of the reflection coefficients are studied. We prove the existence of potential $u(x, t)$ whose scattering data depend on t according to GGKM's formulas. Finally in §4 we show that this function satisfies the KdV equation.