Suzuki, T.
Osaka J. Math.
11 (1974), 37–48

RELATIVE EFFICIENCY OF THE SEQUENCES OF STATISTICAL TESTS

TAKERU SUZUKI

(Received April 7, 1973)

1. Introduction. In this paper giving an extension of the theorem on Pitman efficiency in Noether [2], we try to compare two sequences of tests under more general conditions than Noether [2]. Roughly speaking, the idea of the Pitman efficiency is as follows.

DEFINITION. Given two sequences of tests of the same size of the same statistical hypothesis, the Pitman efficiency of the second sequence of tests with respect to the first sequence is given by the ratio n_1/n_2, where n_2 is the sample size of the second test required to achieve the same power for a given alternative $\theta=\pi_{n_2}(\omega_2)$ as is achieved by the first test with respect to the same alternative $\theta=\pi_{n_1}(\omega_1)$ when the sample size n_1. Here $\pi_n(\omega)$ is a parametric function.

In the paper of Noether [2], it was considered only when (a) the sequence $\{T_n\}$ of statisics is asymptotically normally distributed, (b) the test ϕ_n is such one that $\phi_n=1$ or 0 according as $T_n>c_n$ or $T_n<c_n$ with some constant c_n, and (c) the alternatives $\pi_n(\omega)$ are the following one; $\pi_n(\omega)=\theta_0+n^{-\delta}(\omega-\theta_0)$. In this paper, however, it is shown that the Pitman efficiency is also calculable under more general conditions than those.

In Section 2 we investigate on the rate of convergence of alternatives $\{\pi_n(\omega)\}$. Section 3 is devoted to the calculation of the Pitman efficiency.

2. The rates of convergence of alternatives. Throughout this paper we shall use the following notations. Let Θ be a nonempty subset of R^1 and θ_0 a fixed inner point of Θ. Let K ($\neq\{0\}$) be a fixed cone in R^1, and we denote $\Omega=\{\theta+\theta_0;\ \theta\in K\}(=K+\theta_0)$ and $\Theta_1=\Theta\cap\Omega$. For each $n\in N=\{1,2,\cdots\}$, let $(\boldsymbol{X}_n, \boldsymbol{A}_n)$ be the cartesian product of n copies of a certain measurable space $(\boldsymbol{X}, \boldsymbol{A})$. For each $\theta\in\Theta$ let P_θ be a probability measure on $(\boldsymbol{X}, \boldsymbol{A})$. Let $P_{\theta,n}$ be the product measure of n copies of P_θ. Let a measure space $(\boldsymbol{Y}, \boldsymbol{B}, \mu)$ be given, where $\boldsymbol{Y}$ is a Borel subset of R^r, $\boldsymbol{B}$ is the Borel σ-field in $\boldsymbol{Y}$ and μ is the Lebesgue measure on $(\boldsymbol{Y}, \boldsymbol{B})$.

DEFINITIN 1. Let $\{Q_{\omega,n};\ \omega\in\Omega\}_{n\in N}$ be a squence of families of probability measures and $\{Q_\omega;\ \omega\in\Omega\}$ a family of probability measures on $(\boldsymbol{Y}, \boldsymbol{B})$. Let Ω_0