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MULTIPLY TRANSITIVE PERMUTATION GROUPS AND ODD PRIMES

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In [4] M. Hall determined all 4-fold transtitive permutation groups whose stabilizer of 4 points is of odd order. In this note we give some analogous version of M. Hall's theorem for any odd prime p on $3p$-fold transtive permutation groups. We note that such a version is also already obtained by E. Bannai [1] on (p^2+p)-fold transitive permutation groups.

Theorem. *Let p be an odd prime. Let G be a $3p$-fold transitive permutation group on $\Omega=\{1, 2, \cdots, n\}$. If the order of a stabilizer of $3p$ points in G is prime to p, then $G=S_n$ $(3p\leqslant n<4p)$ or $G=A_n$ $(3p+2\leqslant n<4p)$.*

Our notation follows Nagao [6]. Let us recall some of them: For a set S of permutations on Ω the set of the points left fixed by S will be denoted by $I(S)$. For a permutation x let $\alpha_i(x)$ denote the number of i-cycles. Also let $I^c(S)=\Omega-I(S)$ and $\alpha(x)=\alpha_1(x)$. The order of a permutation x will be denoted by $o(x)$. $p\,|\,o(x)$ will mean that $o(x)$ is divisible by p and $p\not|\,o(x)$ will mean that $o(x)$ is not divisible by p.

1. On $2p$-fold transitive groups

The next lemma which is indebted to Nagao [6] is essential in the present work.

Lemma 1.1. *Let X be a p-fold transitive permutation group on a finite set Ω. Let P be a Sylow p-subgroup of X. If P is semiregular on Ω-$I(P)$, then*

(i) *X has only one conjugacy class of the elements of order p, and*

(ii) *for an element u of order p, $C_X(u)$ is transitive on $I^c(u)$.*

Proof. Since X is p-fold transitive,

$$\frac{|X|}{p}=\sum_{x\in X}\alpha_p(x)\,, \tag{1}$$

by a result of Frobenius [1][2]. On the other hand, since P is semiregular, any element x with p-cycle is uniquely expressed as a product of an element