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MODULES OVER DEDEKIND PRIME RINGS I

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The purpose of this paper is the investigation of modules over Dedekind prime rings. In Section 1, we shall prove that the double centralizer of a P-primary module over a Dedekind prime ring R is isomorphic to $\hat{R}_P$ or $\hat{R}_P/\hat{P}^n$, where P is a nonzero prime ideal of R and $\hat{R}_P$ is the P-adic completion of R with unique maximal ideal $\hat{P}$. Using this result we shall determine the structure of the double centralizer of primary modules over bounded Dedekind prime rings. In Section 2, we shall give a characterization of quasi-injective modules over bounded Dedekind prime rings. This paper is a continuation of [7] and [8]. A number of concepts and results are needed from [7] and [8].

1. The double centralizer of torsion modules

Throughout this paper, R will denote a Dedekind prime ring with the two-sided quotient ring Q, we denote the completion of R with respect to P by $\hat{R}_P$ and its maximal ideal by $\hat{P}$. By Theorem 1. 1 of [6], $\hat{R}_P$ is a complete, g-discrete valuation ring in the sense of [8] and $\hat{R}_P=(\hat{L})_k$, where $\hat{L}$ is a complete, discrete valuation ring with unique maximal ideal $\hat{P}_0$. Further, $\hat{P}=p_0\hat{R}_P=\hat{R}_Pp_0$, where $p_0\in\hat{L}$ with $\hat{P}_0=p_0\hat{L}=\hat{L}p_0$. Since the proper ideals of $\hat{R}_P$ are only the powers of $\hat{P}$, we obtain $\hat{P}^n=\hat{R}_PP^n\hat{R}_P$ for $n=0, 1, 2, \cdots$ (cf. the proof of Theorem 4.5 of [4]). In this section we denote the complete set of the martix units of $\hat{R}_P=(\hat{L})_k$ by e_{ij} $(i, j=1, 2, \cdots, k)$.

Let M be a P-primary module. Then, by the same way as in Lemma 3.14 of [7], M is an $\hat{R}_P$-module by a natural way. It is evident that $\mathrm{Hom}_R(M, M)=\mathrm{Hom}_{\hat{R}_P}(M, M)$ and that M is torsion as an $\hat{R}_P$-module. If M is indecomposable, P-primary and divisible, then M is isomorphic to $\varinjlim e_{11}\hat{R}_P/e_{11}\hat{P}^n$, and we denote it by $R(P^\infty)$. If M is indecomposable, P-primary with $O(M)=P^n$, then M is isomorphic to $e_{11}\hat{R}_P/e_{11}\hat{P}^n$, and we denote it by $R(P^n)$.

Lemma 1.1. *Let R be a Dedekind prime ring. Then the double centralizer D_n of the module $R(P^n)$ is isomorphic to $\hat{R}_P/\hat{P}^n$.*

Proof. By Lemma 3.20 of [7], $L_n=\mathrm{Hom}_R(R(P^n), R(P^n))$, where $L_n=\hat{L}/\hat{P}_0^n$. Hence we have