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PERFECT CATEGORIES IV

(QUASI-FROBENIUS CATEGORIES)

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The author defined perfect Grothendieck categories and studied them [11]. In [12], [13] he developed [11] and determined hereditary perfect categories and hereditary perfect and QF-3 categories.

In this note, as a continuos work we define quasi-Frobenius categories (briefly QF) and generalize some properties of QF-rings.

Let $\mathfrak{A}$ be a Grothendieck category. We always assume $\mathfrak{A}$ contains a generating set $\{G_\alpha\}_I$ of small objects G_α, *e.g.* functor categories. If every projective objects in $\mathfrak{A}$ are injective, we call $\mathfrak{A}$ a QF-category. As we see in examples of QF-categories, some important properties of QF-rings are not inherited to QF-categories.

The object of this paper is to fill those gaps. We assume mainly that G_α's are projective, then QF-categories are perfect. It is clear that all of results in the category $\mathfrak{M}_R$ of modules over a ring R with identity are not valid in perfect categories $\mathfrak{A}$. However, modifying proofs in $\mathfrak{M}_R$, we sometimes succeed to extend some properties in $\mathfrak{M}_R$ to $\mathfrak{A}$. All of theorems in this note are well known in $\mathfrak{M}_R$ and so we shall give often only methods how to modify proofs in $\mathfrak{M}_R$.

In §1 we generalize the notion of Σ-injective [5] and obtain [5], Proposition 3 in $\mathfrak{A}$. We define a QF-category in §2 and generalize results in [4] and [14]. In §3 we deal with a problem whether a QF-category has the following property or not: every injetives are projective, (see [6]). In the final sction, we give some supplementary results of [10].

In this paper, rings S need not to have the identity, unless otherwise stated. We refer the readr to [11], [12] and [13] for notations and definitions.

1. Σ-injective

Let $\mathfrak{A}$ be a Grothendieck category. We always assume that $\mathfrak{A}$ has a generating set $\{G_\alpha\}_I$ of small objects G_α.

1) See [11] and [12] for the definitions.