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PROJECTIVE DIMENSION OF COMPLEX BORDISM MODULES OF CW-SPECTRA, II

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In the previous paper [I] with the same title we tried to extend some results of [6, 8 and 9] to connective *CW*-spectra X. And we gave necessary and sufficient conditions that the Thom homomorphism

$$\mu=\mu\langle 0\rangle: MU_*(X)\rightarrow MU\langle 0\rangle_*(X)\simeq H_*(X)$$

is an epimorphism and that the homomorphism

$$\zeta=\mu_{Td}\langle 1\rangle: MU_*(X)\rightarrow MU_{Td}\langle 1\rangle_*(X)\simeq k_*(X)$$

(lifting the Thom homomorphism $\mu_C: MU_*(X)\rightarrow K_*(X)$) is an epimorphism.

In the present paper we study conditions that

$$\mu\langle n\rangle: MU_*(X)\rightarrow MU\langle n\rangle_*(X)$$

is an epimorphism for a general $n\geqq 0$.

As our main results we have

Theorem 1. *Let X be a connective CW-spectrum and $0\leqq n<\infty$. The following conditions are equivalent*:

I) $\mu\langle n\rangle: MU_*(X)\rightarrow MU\langle n\rangle_*(X)$ *is an epimorphism*;

II) $\mu\langle n\rangle$ *induces an isomorphism* $\tilde{\mu}\langle n\rangle: MU\langle n\rangle_*\otimes_{MU_*} MU_*(X)\rightarrow MU\langle n\rangle_*(X)$;

III) $\mathrm{Tor}^{MU_*}_{p,*}(MU\langle n\rangle_*, MU_*(X))=0$ *for all* $p\geqq 1$;

III)′ $\mathrm{Tor}^{MU_*}_{1,*}(MU\langle n\rangle_*, MU_*(X))=0$.

Theorem 2. *Let X be a connective CW-spectrum and $0\leqq n<\infty$. If one of the equivalent conditions stated in Theorem 1 is satisfied, then*

0) $\mathrm{hom\,dim}_{MU_*}MU_*(X)\leqq n+1$.

We use all notations and notions defined in [I] and quote the theorem of [I] in such a form as "Theorem I. 4".