Yosimura, Z.
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PROJECTIVE DIMENSION OF COMPLEX BORDISM MODULES OF CW-SPECTRA, I

ZEN-ICHI YOSIMURA

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Let $MU_*(\ )$ be the (reduced) complex bordism theory defined on the Boardman's stable category [4] of CW-spectra. Recall that MU_* $(\equiv MU_*(S^0))$ $\cong Z[x_1, x_2, \cdots]$, deg $x_i=2i$. In [3] Baas has constructed a tower of homology theories

$$MU_*(\ ) = MU\langle\infty\rangle_*(\ ) \to\cdots\to MU\langle n\rangle_*(\ ) \to\cdots\to MU\langle 0\rangle_*(\ )\cong H_*(\ )$$

such that $MU\langle n\rangle_*$ $(\equiv MU\langle n\rangle_*(S^0))\cong Z[x_1, \cdots, x_n]$, which factorizes the Thom homomorphism μ: $MU_*(\ )\to H_*(\ )$. When $Td(x_1)=1$ and $Td(x_j)=0$ for all $j\geqq 2$ (it is possible to choose ring generators x_i of MU_* with such properties), we shall write $MU_{Td}\langle n\rangle_*(\ )$ instead of $MU\langle n\rangle_*(\ )$ for emphasis. $MU_{Td}\langle 1\rangle_*(\ )$ can be identified with the connective homology K-theory $k_*(\ )$. Then the tower of homology theories

$$MU_*(\ ) \to\cdots\to MU_{Td}\langle n\rangle_*(\ ) \to\cdots\to MU_{Td}\langle 1\rangle_*(\ ) \cong k_*(\ )$$

factorizes the homomorphism ζ: $MU_*(\ )\to k_*(\ )$ lifting the Thom homomorphism μ_C: $MU_*(\ )\to K_*(\ )$.

Under the assumption that X is a finite CW-complex, Conner, Smith and Johnson ([6] and [9]) investigated conditions that the Thom homomorphism μ: $MU_*(X)\to H_*(X)$ is an epimorphism, and that the homomorphism ζ: $MU_*(X)\to k_*(X)$ is an epimorphism. In the present paper we try to extend these results to a CW-spectrum.

In §1 we study some basic properties of CW-spectra and homology theories $MU\langle n\rangle_*(\ )$ for the sake of our later references.

Landweber [10] indicated that there exists a MU_*-resolution for a CW-spectrum as well as a finite CW-complex (Theorem 1). In §2 we construct two spectral sequences

i) $E\langle n\rangle^2_{p,q}(X)=\mathrm{Tor}^{MU_*}_{p,q}(MU\langle n\rangle_*, MU_*(X))\Rightarrow MU\langle n\rangle_*(X)$

and