CHARACTERISTIC CLASSES WITH VALUES IN COMPLEX COBORDISM

MINORU NAKAOKA

Introduction

This paper is concerned with the characteristic classes for complex bundles with values in the complex cobordism $U^*(\cdot)$. These are the dual Chern classes $\bar{c}^R$, the Wu classes u^R and the classes q corresponding to the power operations P. On these classes with values in the classical cohomology, Haefliger and Wu have proved some interesting theorems in [11], [19], [20]. The aim of this paper is to show the complex cobordism version of their theorems.

Quillen [17] has given a formula relating the power operation P to the Landweber-Novikov operations s^R, and a formula relating the class q to the Chern classes c^R. These formulae play a central role in this paper.

The layout of this paper is as follows.

§1 contains a recall of the Landweber-Novikov operations and the conjugation in Hopf algebras. In §2 we consider the dual Chern classes $\bar{c}^R(\xi)$ and the Wu classes $u^R(\xi)$ of a complex bundle ξ in connection with the Landweber-Novikov operations s^R and their conjugations $\bar{s}^R$. §3 is devoted to the dual Chern classes $\bar{c}^R(M)$ and the Wu classes $u^R(M)$ of a weakly complex manifold M, with which a Riemann-Roch type theorem is proved along the line of Atiyah-Hirzebruch [4]. We have in particular the following formula which may be regarded as a complex cobordism version of the formulae in Wu [19], [20]:

$$\langle s^R \alpha, [M] \rangle = \sum_{I+J=R} s^I \langle \alpha \cdot u^J(M), [M] \rangle ,$$

where $\alpha \in U^*(M)$ and $[M] \in U_*(M)$ is the fundamental class of M.

In §4 and §5, we consider the power operations P and the corresponding characteristic classes q, and give a proof of the formulae due to Quillen.

In §6 an element $\Delta \in U^*(E_G \underset{G}{\times} M^p)$ is defined after Haefliger [11] for a closed almost complex manifold M, where E_G is the universal G-bundle for a cyclic group G of order p (prime). We prove a formula connecting Δ to $u^R(M)$ in terms of P, which may be regarded as a complex cobordism version of Theorem 3.2 in Haefliger [11].