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# ON HOMOGENEOUS REAL HYPERSURFACES IN A COMPLEX PROJECTIVE SPACE

Dedicated to Professor S. Sasaki on his 60th birthday

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The purpose of this paper is to determine those homogeneous real hypersurfaces in a complex projective space $P_n(\boldsymbol{C})$ of complex dimension $n(\geqq 2)$ which are orbits under analytic subgroups of the projective unitary group $PU(n+1)$, and to give some characterizations of those hypersurfaces. In §1 from each effective Hermitian orthogonal symmetric Lie algebra of rank two we construct an example of homogeneous real hypersurface in $P_n(\boldsymbol{C})$, which we shall call a model space in $P_n(\boldsymbol{C})$. In §2 we show that the class of all homogeneous real hypersurfaces in $P_n(\boldsymbol{C})$ that are orbits under analytic subgroups of $PU(n+1)$ is exhausted by all model spaces. In §§3 and 4 we give some conditions for a real hypersurface in $P_n(\boldsymbol{C})$ to be an orbit under an analytic subgroup of $PU(n+1)$ and in the course of proof we obtain a rigidity theorem in $P_n(\boldsymbol{C})$ analogous to one for hypersurfaces in a real space form.

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## 1. Model spaces

In this section we shall state several model spaces in a complex projective space $P_n(\boldsymbol{C})$ with the Fubini-Study metric of constant holomorphic sectional curvature. They are obtained essentially as orbits under the linear isotropy groups of various Hermitian symmetric spaces of rank two. Precisely, let $(\mathfrak{u}, \theta)$ be an effective orthogonal symmetric Lie algebra of compact type. $\mathfrak{u}$ is a compact semisimple Lie algebra and $\theta$ is an involutive automorphism of $\mathfrak{u}$ ([3]). Let $\mathfrak{u}=\mathfrak{k}+\mathfrak{p}$ be the decomposition of $\mathfrak{u}$ into the eigenspaces of $\theta$ for the eigenvalues $+1$ and $-1$, respectively. Then $\mathfrak{k}$ and $\mathfrak{p}$ satisfy $[\mathfrak{k}, \mathfrak{k}]\subset\mathfrak{k}$, $[\mathfrak{k}, \mathfrak{p}]\subset\mathfrak{p}$ and $[\mathfrak{p}, \mathfrak{p}]\subset\mathfrak{k}$.

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