Shima, H.
Osaka J. Math.
10 (1973), 477–493

ON HOMOGENEOUS KÄHLER MANIFOLDS WITH NON-DEGENERATE CANONICAL HERMITIAN FORM OF SIGNATURE (2, 2(n—1))

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(Received October 19, 1972)

We denote by M a connected homogeneous Kähler manifold of complex dimension n on which a connected Lie group G acts effectively as a group of holomorphic isometries, and by K an isotropy subgroup of G at a point o of M. Let v be the G-invariant volume element corresponding to the Kähler metric. In a local coordinate system $\{z_1, \cdots, z_n\}$, v has an expression $v=i^n F dz_1\wedge\cdots\wedge dz_n\wedge d\bar{z}_1\wedge\cdots\wedge d\bar{z}_n$. The G-invariant hermitian form $h=\sum_{i,j}\frac{\partial^2 \log F}{\partial z_i\partial\bar{z}_j}dz_i d\bar{z}_j$ is called the canonical hermitian form of $M=G/K$. It is known that the Ricci tensor of the Kähler manifold M is equal to $-h$. The purpose of this paper is to prove the following:

Theorem 1. *Let $M=G/K$ be a simply connected homogeneous Kähler manifold with non-degenerate canonical hermitian form h of signature* $(2, 2(n-1))$. *Then, if either G is semi-simple or G contains a one parameter normal subgroup, $M=G/K$ is a holomorphic fibre bundle whose base space is the unit disk $\{z\in \boldsymbol{C};\ |z|<1\}$, and whose fibre is a homogeneous Kähler manifold of a compact sime-simple Lie group.*

In the case of $\dim_C G/K=2$, the assumption of Theorem 1 is fulfilled and we have

Theorem 2. *Let $M=G/K$ be a complex two dimensional homogeneous Kähler manifold with non-degenerate canonical hermitian form h of signature* (2, 2). *Then G is semi-simple or G contains a one parameter normal subgroup.*

As an application of these Theorems, we obtain a classification of complex two dimensional homogeneous Kähler manifolds with non-degenerate canonical hermitian form.

1. Let (I, g) be the G-invariant Kähler structure on M, i.e., I is the G-invariant complex structure tensor on M and g is the G-invariant Kähler metric on M. Let $\mathfrak{g}$ be the Lie algebraof all left invariant vector fields on G and let $\mathfrak{k}$ be the subalgebra of $\mathfrak{g}$ corresponding to K. We denote by π the canonical projection