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ON THE EXISTENCE OF SOLUTIONS OF STOCHASTIC DIFFERENTIAL EQUATIONS

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Let B be an N-dimensional Brownian motion. We treat a stochastic differential equation based on B,

$$(\mathrm{I}) \qquad dX(s) = \alpha(X(s))\,dB(s)+\gamma(X(s))\,ds$$

where $\alpha(x)=(\alpha_{ij}(x))$ is an $N\times N$-matrix and $\gamma(x)=(\gamma_i(x))$ an N-dimensional vector. In the case where coefficients are bounded and Borel measurable, Krylov [6] proved the existence of solutions under the condition that α is uniformly elliptic. But in the stochastic control, we consider a stochastic differential equation of the following form,

$$dX(t) = \alpha(B(t),\, X(t))\,dB(t)+\gamma(X(t))\,dt\,.$$

This equation can be regarded as follows,

$$(*) \qquad \begin{cases} dX(t) = \alpha(X(t),\, Y(t))\,dB(t)+\gamma(X(t))\,dt \\ Y(t) = B(t) \end{cases}$$

In this case, the uniform ellipticity of the coefficient of Brownian part is not valid.

The purpose of this article is to seek some weaker conditions of solvability which can be applied to $(*)$. Our result on the equation (I) is the following. Let f be a function on R^N and K a compact subset of R^{N-l}. Let us define ${}^K f(\xi)$ and $|||f|||_{p.\Gamma.K}$ by

$${}^K f(\xi) = \sup_{\eta\in K} |f(\xi,\, \eta)|\,, \quad \xi\in R^l$$

and

$$|||f|||_{p,\Gamma,K} = ||{}^K f||_{L^p(\Gamma)}$$

where Γ is a Borel subset of R^l.

Theorem 1. *Suppose that there exist a non-negative integer l, a positive constant $p(>2l)$ and a non-negative bounded Borel function μ, such that*