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ISOMORPHISMS OF β-AUTOMORPHISMS TO MARKOV AUTOMORPHISMS

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0. Introduction

The purpose of the present paper is to construct an isomorphism which shows the following:

Theorem. *A β-automorphism is isomorphic to a mixing simple Markov automorphism in such a way that their futures are mutually isomorphic.*

Though the state of this Markov automorphism is countable and not finite, we obtain immediately from the proof of the theorem:

Corollary 1. *The invariant probability measure of β-transformation is unique under the condition that its metrical entropy coincides with topological entropy* $\log \beta$.

An extention of Ornstein's isomorphism theorem for countable generating partitions ([2]) shows the following known result (Smorodinsky [5], Ito-Takahashi [3]):

Corollary 2. *A β-automorphism is Bernoulli.*

We now give the definition of β-automorphism and auxiliary notions. Let β be a real number >1.

DEFINITION. A *β-transformation* is a transformation T_β of the unit interval $[0, 1]$ into itself defined by the relation

$$T_\beta t \equiv \beta t \qquad (\bmod 1)\ (0 \leqq t < 1) \tag{1}$$

and by $T_\beta^n 1 = \lim_{t \to 1} T_\beta^n t$.

This transformation has been studied by A. Renyi, W. Parry, Ito-Takahashi et al. Parry [3] showed that there is an invariant probability measure for a