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COMPLEX POWERS OF HYPOELLIPTIC PSEUDO-DIFFERENTIAL OPERATORS WITH APPLICATIONS

Dedicated to Professor Yukinari Tôki on his 60th birthday

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Introduction.

Complex powers of a pseudo-differential operator have been defined by Seeley [15] and Burak [2] for the elliptic case, and defined by Nagase-Shinkai [12] and Hayakawa-Kumano-go [5] for a more general case containing semi-elliptic operators.

In the present paper we shall construct complex powers of a hypoelliptic system of pseudo-differential operators, and apply those powers to the generalized Dirichlet problem and the index theory.

The plan of the paper is as follows. In Section 1 we describe well-known results on the theory of pseudo-differential operators which has been developed in Hörmander [6], [7], Kumano-go [9] and Grushin [4]. In Section 2 the strong (or uniform) continuity and the analyticity of pseudo-differential operators with respect to a parameter are examined by means of their symbols. In Section 3 we construct complex powers P_z of a hypoelliptic system P which belongs to a subclass of Hörmander's in [6], p. 164 (c.f. also Šubin [16]).

Section 4 treats the generalized Dirichlet problem for an operator P which admits complex powers P_z. The Sobolev space $H_{s,P}$ associated with P is defined, and a subspace V of $H_{\frac{1}{2},P}$ is defined as the completion of $C_0^\infty(\Omega)$ in the norm of $H_{\frac{1}{2},P}$ for an open set Ω of R^n. We seek the solution of $Pu=f$ for $f\in L^2(\Omega)$ in the space V. Then, the Lax-Milgram theorem can be applied effectively.

Finally Section 5 is the supplement to the first author's paper [10] where the vanishing theorem of the index is proved when an operator P is slowly varying in the sense of [4] and has complex powers.

We try here to reduce the index theory of a hypoelliptic operator Q of order m to an elliptic operator of order 0 (studied in [4]) when the symbol $\sigma(Q)(x, \xi)$ is equally strong to the symbol $\sigma(P)(x, \xi)$ of an operator P which admits complex powers.

Throughout the present paper we shall treat strict algebras of pseudo-differential operators, and investigate the topology of the symbol class precisely