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CONTRIBUTIONS TO THE THEORY OF INTERPOLATION OF OPERATIONS II

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1. Introduction

In this paper we shall discuss the interpolation of operations on intermediate spaces. Our method is the so-called real method. Our purpose is to treat the critical case which appears in singular integral operators. If we consider for example the Hilbert transform $\tilde{f}$ of function f of the class $L\log^+ L(\pm\infty,\infty)$, $\tilde{f}$ exist a.e. but the only local integrability holds. Then we shall discuss their integral estimation on the whole space.

The intermediate space between two Banach spaces was introduced by W.A.J. Luxemburg [6, 7]. This is defined as follows. Given a topological vector space V and two Banach spaces A_1 and A_2 which are contained and continuously embedded in V. If f is an element of A_i $(i=1, 2)$, we denote its norm by $||f||[A_i]$ $(i=1, 2)$. We shall consider the space A_1+A_2 and introduce in it the norm

$$||f||[A_1+A_2] = inf\,(||g||[A_1]+||h||[A_2])$$

where the infimum is taken over all pairs $g\in A_1$ and $h\in A_2$ such that $f=g+h$, then A_1+A_2 also becomes a Banach space. Since A_1 and A_2 are continuously embedded in V, it is evident that A_1+A_2 is also continuously embedded in V.

In what follows we shall consider totally σ-finite measure space (R, μ) and the space V of equivalent classes of real valued measurable functions on R. The equivalent relation here is that of coincidence almost everywhere. If in V we introduce a topology of convergence in measure on sets of finite measure, V becomes a topological vector space. If we take as the interpolation pair $A_1=L^1_\mu$, $A_2=L^p_\mu(1<p<\infty)$ then these are continuously embedded in V. We shall also consider another measure space (S, ν).

Let us consider operation T which transforms measurable functions on R to those on R. The operation T is called quasi-linear if

(i) $T(f_1+f_2)$ is uniquely defined whenever Tf_1 and Tf_2 are defined and