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ON SOME PARABOLIC EQUATIONS OF EVOLUTION IN HILBERT SPACE

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1. Introduction

Let X and V be two Hilbert spaces such that V is a dense subspace of X with continuous imbedding $V \rightarrow X$. Identifying X with its antidual (= the set of continuous antilinear forms on X) we may consider $V \subset X \subset V^*$ algebraically and topologically where V^* is the antidual of V. As is easily seen V is a dense subspace of V^*. The inner product and norm in X are denoted by (f, g) and $|f|$, and those in V are by $((u, v))$ and $||u||$. For $f \in X$ and $u \in V$, (f, u) is equal to the value at u of f considered as an element of V^*, so we denote the V^*-V duality by (f, u) without causing any confusion. Sometimes we write also (u, f) instead of $\overline{(f, u)}$. The norm in V^* is denoted by $||f||_*$.

Let $a(t; u, v)$, $0 \leqq t \leqq T$, be a family of sesquilinear forms defined on $V \times V$ satisfying the following assumptions:

there exist positive constants M, δ, K and $0 < \rho \leqq 1$ such that

$$|a(t; u, v)| \leqq M||u|| \, ||v|| , \tag{1.1}$$

$$\text{Re } a(t; u, u) \geqq \delta ||u||^2, \tag{1.2}$$

$$|a(t; u, v) - a(s; u, v)| \leqq K |t-s|^\rho ||u|| \, ||v|| \tag{1.3}$$

for any $u, v \in V$ and $t, s \in [0, T]$.

We define the operator $A(t)$ in the following manner;

the element $u \in V$ belongs to $D(A(t))$, the domain of $A(t)$, and $A(t)u = f \in X$ if and only if $a(t; u, v) = (f, v)$ for any $v \in V$.

It is well-known that $-A(t)$ generates an anlytic semigroup of bounded operators in X. We consider the initial value problem of the evolution equation in X

$$du(t)/dt + A(t)u(t) = f(t) , \tag{1.4}$$

*) Part of the contents of this paper was talked by the second author at the Conference on Evolution Equations and Functional Analysis held at the University of Kansas, Lawrence, Kansas in June-July, 1970.