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RADIAL CONVERGENCE OF POISSON INTEGRALS ON SYMMETRIC BOUNDED DOMAINS OF TUBE TYPE

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1. Introduction

Let $\mathcal{D}=\{z\in \boldsymbol{C};\ |z|<1\}$ be the unit disc in $\boldsymbol{C}$ and $\mathcal{B}=\{e^{it};\ -\pi\leq t\leq\pi\}$ the boundary of $\mathcal{D}$. For an integrable function f (In this note a function will always mean a complex valued function) on $\mathcal{B}$ with respect to the normalized measure $\frac{1}{2\pi}dt$ on $\mathcal{B}$, we define the Poisson integral of f by

$$F(z)=\frac{1}{2\pi}\int_{-\pi}^{\pi}f(t)P(z,e^{it})dt \qquad \text{for} \quad z\in\mathcal{D}$$

where

$$P(re^{i\theta},e^{it})=\frac{1-r^2}{1-2r\cos(\theta-t)+r^2} \qquad \text{for} \quad 0\leq r<1$$

and it is called the Poisson kernel of the unit disc $\mathcal{D}$. F is a C^∞-function on $\mathcal{D}$ and it is harmonic on $\mathcal{D}$, that is $\Delta F=0$ for the Laplace-Beltrami operator Δ on C^∞-functions on $\mathcal{D}$ with respect to the Poincaré metric on $\mathcal{D}$.

Then the classical Fatou's theorem asserts that for an integrable function f on $\mathcal{B}$,

$$\lim_{r\uparrow 1}F(re^{i\theta})=f(e^{i\theta})$$

for almost every point $e^{i\theta}$ of $\mathcal{B}$ with respect to the measure $\frac{1}{2\pi}d\theta$.

Now let G be any non-compact connected semi-simple Lie group with finite center, and let K be a maximal compact subgroup of G. Then the homogeneous space G/K is a symmetric space of non-compact type. Let $\mathfrak{g}=\mathfrak{k}+\mathfrak{p}$ be the Cartan decomposition of the Lie algebra $\mathfrak{g}$ of G with respect to the Lie algebra $\mathfrak{k}$ of K. Let $\mathfrak{a}$ be a maximal abelian subspace of $\mathfrak{p}$. Fix an order on $\mathfrak{a}$ and let $\mathfrak{a}^+$ be the positive Weyl chamber of $\mathfrak{a}$ with respect to this order. Let M be the centralizer of $\mathfrak{a}$ in K. Then the homogeneous space K/M is the maximal boundary of G/K in the sense of Furstenberg [2]. Let μ be the normalized