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AN ANALOGUE OF THE PALEY-WIENER THEOREM FOR THE EUCLIDEAN MOTION GROUP

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1. Introduction

The purpose of this paper is to give a detailed proof of an analogue of the Paley-Wiener theorem for the euclidean motion group which was announced in [3]. Restricting our attention to bi-invariant functions (with respect to the rotation group) we obtain an analogue of the Paley-Wiener theorem for the Fourier -Bessel transform.

2. Unitary representations

Let G be the group of all motions of the n-dimensional euclidean space $\boldsymbol{R}^n$. Then G is realized as the group of $(n+1)\times(n+1)$-matrices of the form $\begin{pmatrix} k & x \\ 0 & 1 \end{pmatrix}$, $(k\in SO(n),\ x\in\boldsymbol{R}^n)$. Let K and H be the closed subgroups consisting of the elements $\begin{pmatrix} k & 0 \\ 0 & 1 \end{pmatrix}$, $(k\in SO(n))$ and $\begin{pmatrix} 1 & x \\ 0 & 1 \end{pmatrix}$, $(x\in\boldsymbol{R}^n)$, respectively. Then G is the semi-direct product of H and K. We normalize the Haar measure dg on G such that $dg=dxdk$, where $dx=(2\pi)^{-n/2}dx_1\cdots dx_n$ and dk is the normalized Haar maesure on K.

For any subgroup G_1 of G we denote by $\hat{G}_1$ the set of all equivalence classses of irreducible unitary representations of G_1. For an irreducible unitary representation σ of G_1, we denote by $[\sigma]$ the equivalence class which contains σ. For simplicity we identify $k\in SO(n)$ with $\begin{pmatrix} k & 0 \\ 0 & 1 \end{pmatrix}\in K$ and $x\in\boldsymbol{R}^n$ with $\begin{pmatrix} 1 & x \\ 0 & 1 \end{pmatrix}$ $\in H$. Denote by $\langle\ ,\ \rangle$ the euclidean inner product on $\boldsymbol{R}^n$. Then we can identify $\hat{H}$ with $\boldsymbol{R}^n$ so that the value of $\xi\in\hat{H}$ at $x\in H$ is $e^{i\langle\xi,x\rangle}$. Because H is normal, K acts on H and therefore on $\hat{H}$ naturally: $\langle k\xi, x\rangle=\langle\xi, k^{-1}x\rangle$. Let K_ξ be the isotropy subgroup of K at $\xi\in\hat{H}$. If $\xi\neq 0$, K_ξ is isomorphic to $SO(n-1)$.

The dual space $\hat{G}$ of G was completely determined by G. W. Mackey [4] and S. Itô [2] as follows.

Let $\mathfrak{H}=L_2(K)$ be the Hilbert space of all square integrable functions on K. We denote by U^ξ the unitary representation of G induced by $\xi\in\hat{H}$. Then for