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CHARACTERIZATION OF SLICES AND RIBBONS

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When the definition of ribbon knot was made [2, p. 172], it was with the expectation that it would subsequently be proved that every slice knot is a ribbon knot (the converse being obvious), thereby establishing a simple characterization in 3-dimensional space R^3 of slice knots. Unfortunately this has turned out to be a very difficult thing either to prove or disprove[1]. Although such a 3-dimensional characterization may easily be obtained by suitably modifying the definition of ribbon knot, unless an example of a slice knot that is not a ribbon knot is found this would be somewhat unsatisfactory, because the striking simplicity of the original definition would be lost.

At any rate what I am going to do here is to give new 3-dimensional characterizations of ribbon knots and slice knots. It is hoped that these new characterizations may throw some light on the relationship between ribbon knots and slice knots. In this direction they do lead to an extremely simple derivation of a condition satisfied by the Seifert matrix of a ribbon knot, which condition yields at once all of the known restrictions on the algebraic invariants of a ribbon knot. It also shows that no knot invariants derivable from a Seifert matrix can ever be used to show that a slice knot is not a ribbon knot.

Extending slightly a terminology introduced by Papakyriakopoulos [8, p. 5], let me call a normal singular surface f: $S \rightarrow M$ *canonical* if there are no branch points and the boundary ∂S of S is mapped topologically into M by f. (For simplicity it is assumed that the 3-dimensional manifold M is orientable and that the surface S is compact and orientable.) The singularity of a canonical surface consists of a finite number of triple points and a finite number of double lines which cross themselves and each other at the triple points. Each double line J is one or another of the following three types:

(1) a closed curve whose antecedents are closed curves J' and J'' that lie in the interior $\mathcal{J}S$ of S;

(2) an arc whose antecedents are an arc J' that spans the boundary ∂S of S and

[1] The proof presented in [4] has an error in the second paragraph of p. 380. This fact was communicated to me by the authors, who cited diagram 2 to illustrate the difficulty. Diagram 1, from which diagram 2 may be generated was communicated to me by I. Johansson.