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EQUIVARIANT COHOMOLOGY THEORIES ON G-CW COMPLEXES

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Introduction

G.Bredon developed the equivariant (generalized) cohomology theories in [3], in which he had to restrict himself to the case of finite groups. One of the purposes of this note is to generalize his theory by replacing G-complexes with G-CW complexes. Then, for example, the followings are still true for the case in which G is an arbitrary topological group. The E_2-term of the Atiyah-Hirzebruch spectral sequence associated to a G-cohomology theroy (in this note we frequently use 'G-' instead of 'equivariant') is a classical G-cohomology theory, which is easy to calculate (§1∼§4). The G-obstruction theory works in a classical G-cohomology theory (§5). Moreover, for a G-cohomology theory we get a representation theorem of E.Brown (§6) and the Maunder's spectral sequence (§7).

As an application we study the equivariant K^*-theory in the last session (§8). The Atiyah-Hirzebruch spectral sequence for $K_G^*(X)$ collapses, if dim $X/G \leqq 2$ or X satisfies some other conditions. The E_2-term depends only on the orbit type decomposition of the orbit space, if X is a regular $O(n)$-manifold or the like. These facts enable us to calculate the equivariant K^*-group of Hirzebruch-Mayer $O(n)$-manifolds and Jänich knot $O(n)$-manifolds. Our spectral sequence for a differentiable G-manifold is similar to that of G.Segal which is defined by the equivariant nerve of his [13], but ours is easier to calculate the E_2-term.

In this note G denotes a fixed topological group. Terminologies and notation follow those of [3], [9], [10] in general, though σ denotes a closed cell which is the closure of an (open) cell in the definition of a G-CW complex in [10]. And $G\sigma$ denotes the G-orbit of σ and H_σ the unique isotropy subgroup at any interior point of σ. §0 is exposed for reference to the properties of G-CW complexes.

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