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STRUCTURE PRESERVING GROUP ACTIONS ON STABLY ALMOST COMPLEX MANIFOLDS

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1. Introduction

Conner and Floyd in [1, 2] introduced the notion of periodic maps preserving a complex structure, applying bordism methods quite successfully. In a discussion with Gary Hamrick it became apparent that a somewhat weaker notion was also quite plausible, and the object of this note is to analyze this weaker structure.

Being given a manifold with boundary V and a differentiable action $\phi\colon G\times V\to V$, with G a finite group, the differential $d\phi\colon G\times\tau(V)\to\tau(V)$ induces a G action on the tangent bundle to V. Being given a real representation $\theta\colon G\times W\to W$ of G on a vector space W, one may form a G-bundle $W\times V\xrightarrow{\pi} V$, where G acts by $\theta\times\phi$ on $W\times V$. Then the Whitney sum of $\tau(V)$ and the bundle π has a G-action given by $d\phi$ and θ. Thinking of $E(\tau(V)\oplus\pi)$ as identified with $E(\tau(V))\times W$, the action is $d\phi\times\theta$.

A bundle map $J\colon\tau(V)\oplus\pi\to\tau(V)\oplus\pi$ which covers the identity map on V and such that $J^2=-1$ in the fibers gives $\tau(V)\oplus\pi$ a complex structure and if J commutes with the G action $d\phi\times\theta$, $\tau(V)\oplus\pi$ becomes a complex G-bundle over V.

If $\psi\colon G\times T\to T$ is a complex representation of G one may form the bundle $\bar{\pi}\colon T\times V\to V$ with G action given by $\psi\times\phi$, and if $i\colon T\to T$ is the function with $i^2=-1$giving the complex structure, $\tau(V)\oplus\pi\oplus\bar{\pi}$ is a complex G bundle if G acts by $d\phi\times\theta\times\psi$ and the complex structure is $J\times i$.

A stably almost complex structure on (V, ϕ) preserved by G would then be an equivalence class of systems (W,θ,J), where two such (W,θ,J) and (W',θ',J') are equivalent if there are complex representations (T,ψ,i) and (T',ψ',i') so that $\tau(V)\oplus\pi\oplus\bar{\pi}$ and $\tau(V)\oplus\pi'\oplus\bar{\pi}'$ are equivalent complex G-bundles.

The boundary of V inherits a stably almost complex structure preserved by G for $\tau(\partial V)\cong\tau(V)|_{\partial V}\oplus 1$ as G-bundles, where 1 is the trivial line bundle coming from the trivial representation of G.

It is clear that this differs from the Conner-Floyd approach in which (W,θ) and (T,ψ) are restricted to be trivial representations.