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A NOTE ON THE FORMAL GROUP LAW OF UNORIENTED COBORDISM THEORY

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Introduction

This is a continuation of the author's previous work [6] on the cobordism generators defined by J.M. Boardman in [1]. Previously we have used the Landweber-Novikov operations to calculate the coefficients z_{2i} and z_{4i+1} of a primitive element

$$P=W_1+z_2W_1^3+z_4W_1^5+z_5W_1^6+z_6W_1^7+z_7W_1^8+\cdots$$

in $\mathfrak{N}^*(BO(1))$.

This time we use the Steenrod-tom Dieck operations in the unoriented cobordism theory ([2], [8]) to deduce that the coefficient z_{i-1} for the "canonical primitive element" P_0 is represented by the "iterated Dold manifold" $(R_1)^a(P_{2b})$ for $i=2^a(2b+1)$, where $R_1(M)=S^1\times(M\times M)/a\times T$ (Theorem 3.2).

In other words, let $L=Z_2[e_{i-1}\colon i\neq 2^k]$ be the Lazard ring of characteristic 2 and $F(x, y)=g^{-1}(g(x)+g(y))$ with $g(x)=\sum_{i\geqslant 1}e_{i-1}x^i(e_0=1,\ e_{2^k-1}=0)$ be the universal formal group law. Then the canonical ring isomorphism of Quillen [5] $\varphi\colon L\to\mathfrak{N}^*$ sends the generator e_{i-1} to $[(R_1)^a(P_{2b})]$ for $i=2^a(2b+1)$.

We also study the behaviour of the Dold-tom Dieck homomorphism $R_j\colon \mathfrak{N}_*\to\mathfrak{N}_{2*+j}$ defined by $R_j([M])=[S^j\times(M\times M)/a\times T]$. In particular, we present the following product formula (Lemma 2.2);

$$R_j(xy)=\sum_{j\geqslant k+m\geqslant 0}(\sum\prod_{i\geqslant 0}[P_{2n_i}]^{2^i})R_k(x)R_m(y)\,.$$

In the final section, we examine the relation between the algebra structure of $\mathfrak{N}_*(BO(1))\cong\mathfrak{N}_*(Z_2)$ and the coalgebra structure of $\mathfrak{N}^*(BO(1))$. As an application, we obtain the following formulas for the Smith homomorphism Δ ([3]);

$$\Delta([S^m, a]\cdot[S^n, a])=\sum_{i,j\geqslant 0}a_{i,j}\Delta^i[S^m, a]\Delta^j[S^n, a]$$
$$=(\Delta[S^m, a])\,[S^n, a]+[S^m, a]\,(\Delta[S^n, a])+[P_2](\Delta[S^m, a]\Delta^2[S^n, a]$$
$$+\Delta^2[S^m, a]\Delta[S^n, a])+\cdots,\text{ and}$$
$$\Delta^{2k}([S^m, a]\cdot x)=[S^m, a]\cdot\Delta^{2k}(x)$$