Shimada, N. and Wu, C.-M.
Osaka J. Math.
10 (1973), 25-32

BORDISM ALGEBRAS OF PERIODIC TRANSFORMATIONS

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(Received March 16, 1972)

For the equivariant bordism groups of C^∞-manifolds with differentiable actions of $S^1=U(1)$ and its subgroups Z_n, the cases of free actions have been studied by Conner-Floyd [3], Conner [2], Su [11], Uchida [13], Kamata [5, 6] and others.

The purpose of this note is to study the ring structure of bordism for the cases of semi-free actions (cf. Alexander [1], Miščenko [8]).

The authors express their thanks to the referee for his advice and careful reading of the manuscript.

1. The ring structure of $\mathcal{M}_*(S^i)$ $(i=1, 3)$.

It was shown by Conner-Floyd [3] and Uchida [12] that the following sequences are exact (and also split):

$$(1.1) \qquad 0 \to \mathcal{I}_*(Z_2) \xrightarrow{\nu} \mathcal{M}_*(Z_2) \xrightarrow{\partial} \mathcal{N}_*(Z_2) \to 0\,,$$

$$(1.2) \qquad 0 \to \mathcal{O}_*(S^1) \xrightarrow{\nu} \mathcal{M}_*(S^1) \xrightarrow{\partial} \Omega_*(S^1) \to 0\,,$$

$$(1.3) \qquad 0 \to \mathcal{O}_*(S^3) \xrightarrow{\nu} \mathcal{M}_*(S^3) \xrightarrow{\partial} \Omega_*(S^3) \to 0\,,$$

where $\mathcal{I}_*(Z_2)$ is the bordism group of unoriented manifolds with involution and $\mathcal{O}_*(S^i)$ $(i=1, 3)$ are the bordism groups of oriented manifolds with semi-free S^i-action. Corresponding to these bordsim groups, the cases of free involution and free S^i-action are denoted by $\mathcal{N}_*(Z_2)$ and $\Omega_*(S^i)$ respectively. And $\mathcal{M}_*(Z_2)=\Sigma_{k\geq 0}\mathcal{N}_*(BO(k))$, $\mathcal{M}_*(S^1)=\Sigma_{k\geq 0}\Omega_*(BU(k))$ and $\mathcal{M}_*(S^3)=\Sigma_{k\geq 0}\Omega_*(BSp(k))$.

The above three exact sequences are apparently analogous, and in fact we can study them under a uniform argument.

Let F denote either one of the fields of real numbers R, complex numbers C, or quaternions H. Let $d=\dim_R F$, and let $FP(n)$ denote the n-dimensional projective space.

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*) The second author was supported by the National Science Council of the Republic of China during the preparation of this paper.