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ON THE BIALGEBRAS OF GROUP SCHEMES

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Let G be an algebraic group scheme over an algebraically closed field k. We shall first show that the set $\mathfrak{H}(G)$ of left invariant high order derivations on G will have a natural structure of bialgebra over k with only one grouplike element. If α is a surjective homomorphism of a group variety G onto a group variety G', the kernel H of α in the category of algebraic k-group schemes is well defined. Moreover we have a bialgebra homomorphism $d\alpha$ of $\mathfrak{H}(G)$ into $\mathfrak{H}(G')$. H. Yanagihara showed surjectivity of $d\alpha$ and investigated k-vector space structure of the kernel of $d\alpha$ in the category of bialgebras using the semi-derivations in [13]. In this paper it will be proved that the kernel of $d\alpha$ in the category of bialgebras coincides with the bialgebra of H and we have an exact sequence

$$0 \longrightarrow \mathfrak{H}(H) \longrightarrow \mathfrak{H}(G) \longrightarrow \mathfrak{H}(G') \longrightarrow 0$$

in the category of bialgebras, while the bialgebra of H is not defined in general using the semi-derivations. Thus the bialgebra $\mathfrak{H}(G)$ may be a good substitute of Lie algebras in the case of positive characteristic. The next problem which we are interested is the characterization of sub-bialgebra of $\mathfrak{H}(G)$ which arises from a closed subgroup scheme. Unfortunately we have no general solution, but a solution will be given when G is a commutative group variety over k. Our results have close connection with the work of H. Yanagihara and our bialgebra $\mathfrak{H}(G)$ coincides with the bialgebra used by H. Yanagihara in [12] when G is a group variety.

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1. Local high order derivations of a local ring

Let O be a noetherian local ring containing a field k such that $O/\mathfrak{m}$ is canonically isomorphic to k, where $\mathfrak{m}$ is the unique maximal ideal of O. We denote by $x(o)$ the element of k representing the class of x in O modulo $\mathfrak{m}$. A k-linear homomorphism D of O into k is called a local n-th order derivation of O if we have