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ON THE CONSTRUCTION OF THE LEAST UNIVERSAL HORN CLASS CONTAINING A GIVEN CLASS

Dedicated to Professor Keizo Asano on his 60th birthday

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Many mathematicians have studied universal Horn classes of structures or quasi-primitive (or implicational) classes of algebras. As is well known, these classes have many important algebraic properties which are satisfied in primitive (or equational) classes, for example, to be closed under the formation of direct products, to possess free structures or free algebras if these classes are non-empty. But they are not generally closed under the formation of homomorphic images. Among the theorems with respect to primitive classes, there is the well known most fundamental theorem concerning the construction of the least primitive class containing a given class of algebras. That is, it is obtained by making direct products, subalgebras, and homomorphic images. The main purpose of this paper is to show the analogical theorem which gives the construction of the least universal Horn class containing a given class of structures.

In §1, we simply explain the basic concept and notation with respect to structures for a first order language, and state some well known results which are used in the succeeding sections. In §2, we shall show the definition of a free structure satisfying defining relations, and study the relation between the class K with free structures satisfying defining relations and the formation of substructures and direct products of structures in K. In §3, we shall give the definition of a natural limit structure as a direct limit of a special direct family, and shall show the fact that, a structure $\mathfrak{A}$ is in a universal Horn class K if and only if $\mathfrak{A}$ can be represented as a natural limit structure with respect to K. In §4, applying the results in the above sections, we shall prove the theorem: All isomorphic copies of direct limits of substructures of direct products of structures in a class K form the least universal Horn class containing the class K. Finally, §5 gives a characterization of universal Horn classes by using natural limit structures.

1. Terminologies, notation, and some lemmas

A first order structure or simply structure $\mathfrak{A}$ means a non-empty set A on