BORDISM AND MAPS OF ODD PRIME PERIOD

CHING-MU WU*

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1. Introduction

Let X be a topological space with $A \subset X$ a subspace, and let $\tau \colon (X, A) \to (X, A)$ be a continuous map of period p, p an odd prime. We define oriented equivariant bordism groups with maps of preiod p which are analogues of the equivariant bordism groups of involutions given by Stong [6]. As a special case we obtain Z_p-bordism groups defined by Conner and Floyd [2].

Our aim is to compute such bordism groups and to catch a clearer view of their structures.

The main results of this paper are as follows.

In §2 we define (free) oriented equivariant bordism groups $\mathcal{O}_*(X, A, \tau)$ $(\Omega_*(X, A, \tau))$ and another bordism group $\mathcal{M}_*(X, A, \tau)$, a generalization of the bordism groups $\mathcal{M}_* = \sum \mathcal{N}_m(BO(*-m))$ of involutions given by Conner and Floyd in [2,28.1]. And we obtain

Theorem 1. *The sequence*

$$\cdots \to \Omega_n(X, A, \tau) \xrightarrow{i_*} \mathcal{O}_n(X, A, \tau) \xrightarrow{\nu} \mathcal{M}_n(X, A, \tau) \xrightarrow{\partial} \Omega_{n-1}(X, A, \tau) \to \cdots \text{ is}$$

exact, where i_* forgets freeness, ν is defined by taking the normal disk bundle of the fixed point sets and ∂ is defined by taking boundary.

As a special case we obtain an exact sequence

$$0 \to \Omega_* \xrightarrow{i_*} \mathcal{O}_*(Z_p) \xrightarrow{\nu} \mathcal{M}_*(Z_p) \xrightarrow{\partial} \tilde{\Omega}_*(Z_p) \to 0 .$$

The Ω-modules $\mathcal{M}_*(Z_p)$ and $\mathcal{O}_*(Z_p)$ may be given ring structure, and in this sequence we see that $\mathcal{J} = \operatorname{im} i_*$ is an ideal of $\mathcal{O}_*(Z_p)$. We then have

Corollary 1. 2. *Let $\hat{\mathcal{O}}_*(Z_p) = \mathcal{O}_*(Z_p)/\mathcal{J}$. Then the sequence*

$$0 \to \hat{\mathcal{O}}_*(Z_p) \xrightarrow{\nu} \mathcal{M}_*(Z_p) \xrightarrow{\partial} \tilde{\Omega}_*(Z_p) \to 0$$

is exact.