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# BRAUER GROUPS OF ALGEBRAIC FUNCTION FIELDS AND THEIR ADÈLE RINGS

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**Introduction.** Let $K$ be an algebraic number field and $\{\mathfrak{p}\}$ be the valuations of $K$, then related to Takagi-Artin's class field theory, the following exact sequence is well-known (c.f. Hasse [5]);

$$0 \to Br(K) \to \bigoplus_{\mathfrak{p}} Br(K_{\mathfrak{p}}) \to \boldsymbol{Q}/\boldsymbol{Z} \to 0 \tag{1}$$

where $K_{\mathfrak{p}}$ is the completion of $K$ with respect to $\mathfrak{p}$. In the Seminar 1966 at Bowdoin College, G. Azumaya [4] showed that the middle term of (1) is isomorphic to the Brauer group of the adèle ring $A_K$ of $K$ and that the following diagram with canonical arrows is commutative;

$$\begin{array}{ccccc} & & \bigoplus_{\mathfrak{p}} Br(K_{\mathfrak{p}}) & & \\ & \nearrow & & \searrow & \\ 0 \to Br(K) & & \wr\Vert & & \boldsymbol{Q}/\boldsymbol{Z} \to 0 \\ & \searrow & & \nearrow & \\ & & Br(A_K) & & \end{array} \tag{2}$$

But on an algebraic function field, the class field theory does not hold except the case of finite constant field (Artin-Whalpe [1]), so the analogies of (1), (2) must have fallen.

The purpose of this paper is to clarify the relations of the Brauer group of the adèle ring of a function field, to the Brauer group of a function field and to Galois cohomologies.

We use the following notations:

$k$ : a perfect field
$\bar{k}$ : the algebraic closure of $k$
$F$ : an algebraic function field of one variable over $k$ i.e. $F/k$ is finitely generated, $k$ is algebraically closed in $F$ and the degree of transcendency of $F/k$ is one
$\bar{F}=F\cdot\bar{k}$: the field theoretic compositum of $F$ and $\bar{k}$
$\mathfrak{p}$ : a prime divisor of $F$ over $k$
$F_{\mathfrak{p}}$ : the completion of $F$ with respect to $\mathfrak{p}$
$\mathfrak{O}_{\mathfrak{p}}$ : the valuation ring of $F_{\mathfrak{p}}$