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HYPERSURFACES WITH PARALLEL RICCI TENSOR

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0. Introduction

The purpose of this paper is to classify those Riemannian manifolds with parallel Ricci tensor which arise as hypersurfaces in real space forms. H. B. Lawson, Jr. [1] performed this classification under the assumption of constant mean curvature. Lawson's result may be divided into two parts-determination of the local geometry on the hypersurface, and a rigidity theorem.

In the following, we prove that no assumption on the mean curvature is necessary unless the dimension is 2 or the hypersurface and the ambient space have the same constant curvature. See Theorem 10.

1. The standard examples

We consider first some special complete hypersurfaces which will serve as models in our discussion. $\tilde{M}$ is the ambient space, M is the hypersurface and f: $M\rightarrow\tilde{M}$ is an isometric immersion. In each of the examples, M is a submanifold of $\tilde{M}$ and f is the inclusion mapping.

For $\tilde{M}=E^{n+1}$, we have as our model hypersurfaces, hyperplanes, spheres, and cylinders over spheres.

For $\tilde{M}=S^{n+1}(\tilde{c})$, we have great spheres, small spheres, and products of spheres. The latter may also be thought of as the intersection of two cylinders over spheres in E^{n+2}.

All of the above are explicitly written out in [2] together with their second fundamental forms. We consider the real hyperbolic space of curvature $\tilde{c}<0$ (which we denote by $H^{n+1}(\tilde{c})$) in more detail here since the analogous facts are omitted from [2].

For vectors X and Y in R^{n+2}, we set $g(X, Y)=\sum_{1=1}^{n+1} X^i Y^i\text{-}X^{n+2}Y^{n+2}$. For given $\tilde{c}<0$, we define $R=\frac{1}{\sqrt{-\tilde{c}}}$. Then

$$H^{n+1}(\tilde{c}) = \{x\in R^{n+2}\,|\,g(x, x) = -R^2 \quad\text{and}\quad x_{n+2}>0\}$$

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