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ON BOARDMAN'S GENERATING SETS OF THE UNORIENTED BORDISM RING

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Introduction

For a pointed finite CW pair (X, A), define as usual the k-dimensional unoriented cobordism group $\mathfrak{N}^k(X, A)$ of (X, A) by

$$\mathfrak{N}^k(X, A) = \varinjlim_n [S^{n-k}(X/A), MO(n)]\,,$$

and denote $\sum_{-\infty<k<\infty} \mathfrak{N}^k(X, A)$ by $\mathfrak{N}^*(X, A)$.

We identify the coeficient ring $\mathfrak{N}^*$ with the unoriented bordism ring $\mathfrak{N}_*$ by the Atiyah-Poincaré duality [2]

$$D: \mathfrak{N}_k \to \mathfrak{N}^{-k}\,.$$

Let P_n be the n-dimensional real projective space and η_n be the canonical line bundle over P_n. Define

$$\mathfrak{N}^*(BO(1)) = \varprojlim_n \mathfrak{N}^*(P_n) \cong \mathfrak{N}_*[[W_1]]\,,$$

where $W_1 = \varprojlim_n W_1(\eta_n)$ is the cobordism first Stiefel-Whitney class [4]. On account of the Kunneth formula, the homomorphism

$$\mu^*_{m,n}: \mathfrak{N}^*(P_{m+n}) \to \mathfrak{N}^*(P_m \times P_n)$$

induced by a continuous map $\mu_{m,n}$ satisfying $\mu^*_{m,n}\eta_{m+n} \cong \pi_1^*\eta_m \otimes \pi_2^*\eta_n$ gives rise to the comultiplication

$$\mu^*: \mathfrak{N}^*(BO(1)) \to \mathfrak{N}^*(BO(1)) \underset{\mathfrak{N}_*}{\otimes} \mathfrak{N}^*(BO(1))\,.$$

Let

$$P = W_1 + z_2 W_1^3 + z_4 W_1^5 + z_5 W_1^6 + z_6 W_1^7 + z_7 W_1^8 + \cdots \quad (z_i \in \mathfrak{N}_i)$$

be a primitive element in $\mathfrak{N}^*(BO(1))$ with respect to this comultiplication. Such