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DIFFERENTIAL HOPF ALGEBRAS MODELLED ON K-THEORY MOD p. I

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Introduction

In the present work the authors study properties of Hopf structures modelled on complex K-theory mod p of H-spaces. Namely, let X be an H-space which has a homotopy type of finite CW-complexes, and p a prime. For each choice of the admissible external multiplication μ_p of $K^*(\ ;Z_p)$ [2], $K^*(X;\ Z_p)$ gains a structure of algebra as well as that of coalgebra, hence a kind of structure like Hopf algebra. We study structures modelled on these structures. Our results were already partly announced in [3] and also reported in Neuchâtel conference on H-spaces, 1970 (to appear in Springer Lecture Notes Series) by the first named author.

When we compare our structures like Hopf algebras with the classical Hopf algebras modelled on the ordinary homology and cohomology of H-spaces, we will find two significant differences. The first point of difference is that the classical Hopf algebras are non-negatively graded and can be discussed sometimes by making use of an induction argument on degrees (cf., [10] etc.), but our structures are Z_2-graded and we cannot use such arguments. We use instead sometimes two filtrations (F-filtrations by algebra structure and G-filtrations by coalgebra structure) originally due to Browder [6], or some other arguments.

The second point is that in the classical Hopf algebras the relation

$$\psi\varphi=(\varphi\otimes\varphi)(1\otimes T\otimes 1)(\psi\otimes\psi)$$

of Milnor-Moore [10] is important. But in our structures the above relation may not hold in general. The above relation in the classical case is essentially based on the *commutativity of external multiplications* of ordinary homology and cohomology. But the external multiplication μ_p of $K^*(\ ;Z_p)$ may not be commutative in general, and is never commutative in case $p=2$ [2]. Fortunately the deviation formula from the commutativity is known [2]. So we regard this non-commutativity as a kind of commutativity relation replacing the ordinary twisting morphism T by the λ-modified one T_λ, (2.17). Actually we find the above relation holds also in our structures if we replace T by a suitable T_λ. In the definition of T_λ a differential comes in. Thus we talk of the differential Hopf algebras (in a modified sense) from the beginning.