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ON MULTIPLY TRANSITIVE GROUPS X

Dedicated to Professor Keizo Asano on his 60th birthday

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1. Introduction

In this paper we shall prove the following theorems.

Theorem 1. *Let G be a permutation group on $\Omega=\{1, 2, \cdots, n\}$ where $n>4$. Assume that a Sylow 2-subgroup P of the stabilizer of any four points in G satisfies the following two conditions:*

(i) *P is a nonidentity semi-regular group.*

(ii) *P fixes exactly r points.*

Then

(I) *If $r=4$, then $|\Omega|=6$, 8 or 12, and $G=S_6$, A_8 or M_{12} respectively.*

(II) *If $r=5$, then $|\Omega|=7, 9$ or 13. In particular, if $|\Omega|=9$, then $G\leqq A_9$, and if $|\Omega|=13$, then $G=S_1\times M_{12}$.*

(III) *If $r=7$ and $N_G(P)^{I(P)}\leqq A_7$, then $G=M_{23}$.*

In a previous paper [10] we proved that if G is a 4-fold transitive group and a Sylow 2-subgroup P of a stabilizer of four points in G is not the identity, then P fixes exactly four, five or seven points. Therefore the following corollary is an immediate consequence of Theorem 1.

Corollary. *Let G be a 4-fold transitive group on Ω and assume that a Sylow 2-subgroup P of a stabilizer of four points in G is not the identity. For a point t of $\Omega-I(P)$, assume that a Sylow 2-subgroup R of the stabilizer of any four points in $N_G(P_t)^{I(P_t)}$ satisfies the following two conditions:*

(i) *R is a nonidentity semi-regular group.*

(ii) $|I(R)|=|I(P)|$.

Then one of the conclusions in Theorem 1 *holds for $N_G(P_t)^{I(P_t)}$. In particular, if t is a point of a minimal P-orbit, then $N_G(P_t)^{I(P_t)}$ satisfies the conditions* (i) *and* (ii).

The last assertion of this corollary follows from Lemma 1 of [9].

By using these theorems we have the following