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FOURIER SERIES OF SMOOTH FUNCTIONS ON COMPACT LIE GROUPS

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Introduction

The purpose of the present note is to give an elementary proof of the following theorems. Any C^{2k}-function on a compact connected Lie group G can be expanded by the absolutley and uniformly convergent Fourier series of the matricial components of irreducible representations if $2k > \frac{1}{2}\dim G$ (Theorem 1). The Fourier transform is a topological isomorphism of $C^\infty(G)$ onto the space $S(D)$ of rapidly decreasing functions on the set D of the classes of irreducible representations of G (Theorem 3 and 4).

The related results which the author found in the literature are as follows. In Séminaire Sophus Lie [1] exposé 21, it was proved that any C^∞-functions on G can be expanded by the uniformly convergent Fourier series. Zhelobenko [3] proved Theorem 3 for the group SU(2). R.A. Mayer [4] proved that the Fourier series of any C^1-function on SU(2) is uniformly convergent but there exists a C^1-function on SU(2) whose Fourier series does not converge absolutely.

1. The Fourier expansion of a smooth function

Throughout this paper we use the following notations. G: a compact connected Lie group, G_0: the commutator subgroup of G, T: a maximal toral subgroup of G, l: the rank of $G = \dim T$, p: the rank of G_0, n: the dimension of $G = l + 2m$, $\mathfrak{g}$: the Lie algebra of G, $\mathfrak{g}^c$: the complexification of $\mathfrak{g}$, $\mathfrak{t}$: the Lie algebra of T, R: the root system of $\mathfrak{g}^c$ with respect to $\mathfrak{t}^c$, dg: the Haar measure on G normalized as $\int_G dg = 1$, $L^2(G)$: the Hilbert space of the complex valued square integrable functions on G with respect to dg, $C^k(G)$: the set of all k-times continuously differentiable complex valued functions on G, $||A|| = \mathrm{Tr}(AA^*)^{1/2}$: the Hilbert-Schmidt norm of a matrix A.

In this paper, a finite dimensional continuous matricial representation of G

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