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Osaka, J. Math.
8 (1971), 29-32

# A NOTE ON THE DEFINING EQUATION OF A TRANSITIVE LIE GROUP

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(Received August 17, 1970)

We prove the following theorem: The operations of a transitive Lie group $G$ acting on a manifold $M$ are characterized as solutions of a differential equation on $M$.

**1. Introduction.** Let $G$ be a connected Lie group acting differentiably on a $C^\infty$-differentiable manifold $M$. We assume that the action is transitive. Fix a point $o$ in $M$. By $D^k(o;M)$ we denote the space of all $k$-jets of local diffeomorphisms with source $o$ and target anywhere in $M$. Let $H^k$ be the subset of $D^k(o;M)$ consisting of all $k$-jets with target $o$. Then $H^k$ is a Lie group. The space $D^k(o;M)$ is a principal fiber bundle with base $M$ and structural group $H^k$.

Let $K^o$ be the isotropy subgroup of $G$ at $o$ and $G^k$ be the set of all $k$-jets of actions of $K^o$ with source $o$. Then $G^k$ is a Lie subgroup of $H^k$ (Proposition 1). Let $P^k$ be the set of all $k$-jets of actions of $G$ with source $o$. Then $P^k$ is an associated fiber bundle with fiber $G^k$ to the principal fiber bundle $G(M, K^o)$. Also $P^k$ is a reduced bundle with structural group $G^k$ of the principal fiber bundle $D^k(o:M)$.

Let $P^k(M)$ be the space of all $k$-jets of actions of $G$ with source and target anywhere in $M$. Then $P^k(M)$ is an associated fiber bundle with fiber $P^k$ to the principal fiber bundle $G(M, K^o)$.

**Theorem.** *There exists an integer, $l$, such that the following holds: Suppose $f$ is a local diffeomorphism of $M$ defined on a connected domain $V$. Then $f$ is a restriction of the action of an element $g$ in $G$ to $V$ if and only if $j^l_x(f) \in P^l(M)$ for all $x$ in $V$.*

REMARK 1. Our theorem was stated in a classical form by Lie in [5] for a Lie algebra of vector fields and proved by E. Cartan in [1] for a local Lie group of transformations.

REMARK 2. For a pseudo-group of infinite dimension, Kuranishi [4] gave a sufficient condition in order that it may be defined by a partial differential equation. Also for an infinite dimensional Lie algebra of vector fields, Singer and