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A NOTE ON THE CAPACITY OF RECURRENT MARKOV CHAINS

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1. Introduction

Let P be an irreducible recurrent transition function on a denumerable space S with strictly positive invariant measure α. For a kernel A, a function f and a measure μ on S, we define $Af(x)=\sum_y A(x, y)f(y)$, $\mu A(x)=\sum_y \mu(y)A(y, x)$, $\mu\cdot f=\sum_y \mu(y)f(y)$ and $\mu\cdot 1=\sum_y \mu(y)$. A kernel A on S is called a *weak potential kernel* if Af is bounded and satisfies $(P-I)Af=f$ for all null charge f. A left (right) equilibrium potential for a weak potential kernel A and a set E is the potential $\nu=\mu A$ $(g=Af)$ satisfying $\mu=0$ $(f=0)$ on $S-E$, $\mu\cdot 1=1$ $(\alpha\cdot f=1)$ and $\nu=$constant $\times\alpha(g=$constant$)$ on E. The constant is denoted by $C(E)$ $(C^*(E))$ and is called the *left* (*right*) *capacity* of the set E with respect to (α, A). Its charge $\mu(f)$ is called the *left* (*right*) *equilibrium charge*. The existence of the equilibrium charge and various properties concerning its capacity were discussed in [3], [5] and [6].

In this paper we shall be concerned with the probablistic representation of the equilibrium charge and its capacity for some weak potential kernel. The argument depends on the notion of the approximate chain introduced by Hunt [2]. For a given transient transition function Q on S, a random chain (X, a, b) on a σ-finite measure space $(\Omega, \boldsymbol{B}, \boldsymbol{P})$ is called an *approximate Q-chain* if for every finite set E, (X, a, b) is reduced to a Q-chain by the hitting time σ_E of (X, a, b) for E and satisfies $\boldsymbol{P}[\sigma_E=-\infty]=0$. As was remarked by Hunt, this definition is equivalent to his original definition. In the following the approximate chains are denoted by (X, a, b) and distinguished only by the measure $\boldsymbol{P}$. Particularly if $a(\omega)=0$ a.e. and $\boldsymbol{P}[X_0=z]=I(x, z)$ then we shall use $\boldsymbol{P}_x$ in place of $\boldsymbol{P}$. Moreover the hitting time of (X, a, b) for a finite set E is denoted by σ_E. It is known that for any Q-excessive measure η, there corresponds an approximate Q-chain on $(\Omega, \boldsymbol{B}, \boldsymbol{P})$ satisfying $\eta(x)=\boldsymbol{E}[\sum_{a(\omega)\leq n\leq b(\omega)} I_{\{x\}}(X_n(\omega))]$ where I_E is the indicator function of the set E and $\boldsymbol{E}$ is the expectation with respect to $\boldsymbol{P}$. We shall call (X, a, b) $(\boldsymbol{P})$ the approximate Q-chain (measure) *canonically associated* with η. It was shown by T. Watanabe [4] that the transient capacity,