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ON LINEAR GRAPHS IN 3-SPHERE

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0. Introduction

Throughout this paper we work in the piecewise-linear category, consisting of simplicial complexes and piecewise-linear maps. By $(P\subset M)$ we denote a pair of complexes such that M has an arbitrary but fixed orientation if M is orientable and P is embedded as a subcomplex in M. $\boldsymbol{K}$ denotes a set of all connected finite 1-dimensional complexes. Then, for $K\in\boldsymbol{K}$ we will call $(K\subset S^3)$ a *linear graph,* or simply *graph,* in a 3-dimensional sphere S^3.

The purpose of the paper is to classify $\{(K\subset S^3)\,|\,K\in\boldsymbol{K}\}$ by an equivalence relation, which we will call a neighborhood-congruence. We will introduce a operation $\vee$ of composition in $\{(K\subset S^3)\,|\,K\in\boldsymbol{K}\}$ so that neighborhood-congruence classes of graphs form a commutative semi-group, and give the following as generalization of knots [14] and links [8].

Theorem 3.12. *In the semi-group of all neighborhood-congruence classes of linear graphs, factorization is unique.*

As an immediate application we can discribe socalled knotted solid tori of genus n in the 3-sphere S^3.

1. Definitions and notations

Throughout the paper, ∂M and ϑM denote the boundary and the interior of a manifold M, respectively. For a pair $(P\subset M)$, by $N(P; M)$ we denote a regular neighborhood of P in M, that is, we construct its second derived and take the closed star of P, see [9] and [12]. For any non-negative integer n, $\boldsymbol{K}(n)$ denotes a set of all connected finite 1-dim. complexes whose 1-dim. Betti number is n.

First let us explain an usual equivalence of pairs, see [2], [6].

1.1. DEFINITION. Two pairs $(P\subset M)$ and $(P'\subset M')$ are *congruent* iff there is a homeomorphism $h:M\rightarrow M'$ such that $h(P)=P'$ and h is orientation preserving if M is oriented.