Kori, T.
Osaka J. Math.
5 (1968), 49-68

ON A CHARACTERIZATION OF CERTAIN ADDITIVE FUNCTIONALS OF MARKOV PROCESSES

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(Received February 1, 1968)

0. Introduction

The problem of characterizing positive additive functionals of Markov processes by means of their expectation (called characteristic) has been studied by Volkonsky [5], Meyer [Part II of 3], and Motoo and S. Watanabe [1]. In the present paper we shall give a characterization of square-integrable additive functionals with expectation zero.

Our method is different from, and more elementary than, those of Meyer, Motoo and S. Watanabe; it is a version of the method adopted by Meyer [Part I of 3, 4] in the study of absolute continuity of two Markov processes. Our method is also used for characterizing almost additive functionals without assuming the strong Markov property.

Let $\mathbf{X}=(x_t, P_x, x\in S)$ be a Markov process with a Markov transition function ($P_t(x, S)=1$). Let A be a locally integrable (not necessarily positive) almost additive functional of $\mathbf{X}$, and let us define a system of kernels(1) $(Q_t(x, dy))$ on S by

$$Q_t(x, E) = E_x(A_t\cdot 1_E(x_t))\,, \tag{0.1}$$

where 1_E is the indicator of the set E. $Q_t(x, dy)$ is absolutely continuous with respect to the transition function $P_t(x, dy)=P_x(x_t\in dy)$, and the following equation, called the *characteristic equation*, holds;

$$P_sQ_t+Q_sP_t = Q_{s+t} \qquad \text{for any} \quad s,\ t\geqq 0\,. \tag{0.2}$$

If a system of kernels $(Q_t(x, dy);\ t\geqq 0)$ on S satisfies the characteristic equation and if each $Q_t(x, dy)$ is absolutely continuous with respect to $P_t(x, dy)$ we will call it *a system of characteristic kernels*. The density of $Q_t(x, dy)$ with

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(1) A map $k(x, dy)$ from $S\times \boldsymbol{F}(S)$ to $(-\infty, \infty]$ is called a *kernel* if it satisfies the following properties:
(i) For each $E\in\mathbf{F}(S)$, $k(\cdot, E)$ is a $\mathbf{F}(S)$-measurable function on S.
(ii) For each $x\in S$, $k(x, \cdot)$ is a measure on $\mathbf{F}(S)$ with a finite total variation.