

ON MARKOV CHAINS SIMILAR TO THE REFLECTING BARRIER BROWNIAN MOTION

TOKUZO SHIGA AND TAKESEI WATANABE

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1. Introduction

The reflecting barrier (r. b.) Brownian motion in *one dimension* has been well known [12], [16]. In *higher dimensions*, the r. b. Brownian motion on a bounded domain D with *smooth boundary* ∂D is more or less classical (at least analytically) [13], [17], [18]; its transition density is given by the fundamental solution of the heat equation

$$(1.1) \quad \frac{\partial u}{\partial t} = \frac{1}{2} \Delta u \quad \text{in } D$$

with the boundary condition

$$(1.2) \quad \frac{\partial u}{\partial n} = 0 \quad \text{on } \partial D,$$

where Δ is the Laplacian and $\partial/\partial n$, the normal derivative at the boundary. Recently, Fukushima [9] has constructed the r.b. Brownian motion on an *arbitrary bounded domain* in higher dimensions by solving a functional equation in a Hilbert space which is equivalent to (1.2) if ∂D is smooth but which involves neither ∂D nor $\partial/\partial n$.