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Osaka J. Math.
4 (1967), 233-242

ON SEPARABLE ALGEBRAS OVER A COMMUTATIVE RING*

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(Received August 19, 1967)

Introduction. The notion of a separable algebra over a commutative ring was introduced in Auslander-Goldman [2], which coincides with that of a maximally central algebra in Azumaya [3] for a central algebra over a local ring. The basic properties of separable algebras were shown in [2] and [3].

The purpose of this paper is to define the reduced trace and norm of a central separable algebra over a commutative ring and to prove that a separable algebra over a commutative ring is a symmetric algebra.

Let Λ be a central separable algebra over a commutative ring R and let S be a commutative R-algebra such that $S\otimes_R\Lambda\cong\mathrm{Hom}_S(P, P)$ for some finitely generated, faithful, projective S-module P. Then S is called, according to [2], a *splitting ring* of Λ, and especially, if $R\subseteq S$, it is called a *proper splitting ring* of Λ. It was proved in [2] that a central separable algebra over a Noetherian local ring R has a proper splitting ring which is a Galois extension of R. However, for a general commutative ring R, it is an open problem whether any central separable R-algebra has a proper (Galois) splitting ring. Therefore, our method, which will be used to defining the reduced trace and norm of a central separable R-algebra, is different from the usual one in the classical case (cf. [4]).

In § 1 we shall show that a separable algebra over a general commutative ring is extended from a separable algebra over a Noetherian commutative ring, and, in § 2, we shall prove that, in case R is a commutative ring included in a semi-local ring, a central separable R-algebra has a proper splitting ring.

§ 3 is devoted to defining the reduced trace of a central separable R-algebra Λ. If Λ has a proper splitting ring, we can define the reduced characteristic polynomial, trace and norm of Λ by using the characteristic polynomial, trace and norm of a projective module in [7], and we shall also show that there exist the analogous relations to the classical case between these and the characteristic polynomial, trace and norm of an R-algebra Λ. In the general case, we define the reduced trace of Λ, by using the above-mentioned result in § 1.

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* This work was supported by the Matsunaga Science Foundation.